Approximations Algorithms

(for Database Researchers)

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Optimization and computation problems

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Inconsistent Data Repair. Given a database inconsistent w.r.t. fixed integrity constraints, find the minimum amount of tuples to add or remove to make it consistent.

Influence Maximization. Given a social network with influence probabilities on edges, find the set of nodes to target to maximize the impact of a marketing campaign.

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Probabilistic Query Evaluation. Compute the probability of a fixed SQL query over a database whose tuples are annotated with probabilities.

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Polynomial-time	NP-hard	#P-hard
Max. Matching Nav. XPath Counting	Set Cover Vertex Cover Incons. Data Repair Influence Maximization	Coloring Counting SQL Match Counting Prob. Query Evaluation

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  - Find heuristic algorithms that are good enough in practice, though without any guarantee
  - Find deterministic algorithms that provide a guaranteed approximation
  - Find randomized algorithms that provide a guaranteed approximation with high probability

Outline

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NP-hardness

- A decision (i.e., yes/no) problem is in NP if there exists a nondeterministic polynomial-time algorithm (i.e., the algorithm is allowed to make a guess) that solves it

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Technical note: most definitions of NP-hardness for decision problems are a bit stricter because they are based on Karp many-one reductions. Important if you want to distinguish, e.g., NP-hardness vs coNP-hardness. For optimization/computation problems, it is irrelevant, so I use simpler Turing reductions.

#P-hardness

- A counting problem is in #P if it can be solved by counting the number of ways a nondeterministic polynomial-time algorithm (i.e., the algorithm is allowed to make a guess, and you count the various ways to guess) can return “yes”

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- But Y reduces to Z: use Z to count, and return “yes” iff the count is > 0.
- Therefore Y reduces to X, and thus X is NP-hard.

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Outline

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Additive (absolute) approximation

Let $\varphi : \mathbb{R} \to \mathbb{R}^+$.

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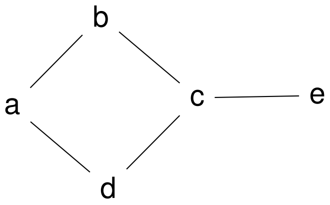
- Additive approximation is thus rarely used, “approximation algorithm” usually means multiplicative
- APX: class of optimization problems that have a polynomial-time multiplicative approximation algorithm with constant φ

Vertex Cover is in APX



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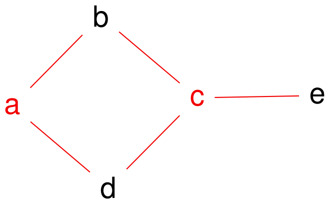

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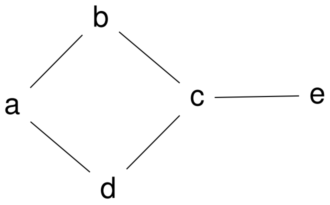

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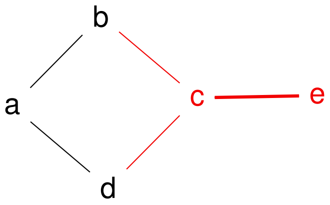

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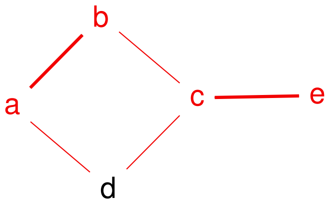

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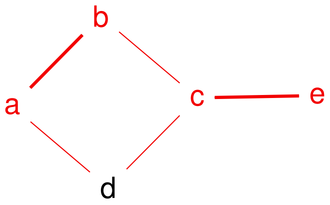

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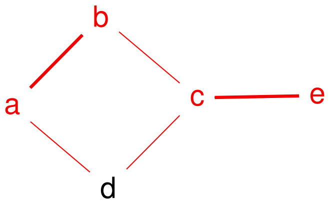

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Multiplicative 2-approximation! (twice as many nodes in the approximated cover as edges chosen, each of this edge need to be covered)

Other examples

- Set Cover has a $(\ln n + O(1))$-approximation algorithm [Chv79] but is not in APX [LY94]
- Inconsistent Data Repair is in APX (but the constant depends on the dependencies) [KL09]
- Influence Maximization is in APX; it has a $(1 - 1/\mathrm{e} - \varepsilon)$-approximation algorithm for any ε (slightly better than 63%) [KKT03]

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- Vertex Cover is not 1.3606-approximable [DS05] (!)
- Set Cover is not $(\ln(n) - o(\ln n))$-approximable [DS14]
- This kind of results is usually much more difficult to obtain than approximation algorithms

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- By exploiting approximation-preserving reductions between a problem and an approximable problem (in both directions); various notions of approximation-preserving, arbitrary reductions don't work

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- FPTAS (Fully Polynomial-Time Approximation Scheme): PTAS whose overall complexity depends polynomially in $1/\varepsilon$

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- Knapsack is an NP-hard problem, but there exists an FPTAS

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Outline

Introduction

Intractable Classes

Deterministic Approximations

Randomized Approximations

- Generalities
- Monte-Carlo Sampling
- FPRAS

Conclusion

Outline

Randomized approximations

To simplify, only talk about computation problems. $\varphi : \mathbb{R} \to \mathbb{R}^+$, $\delta > 0$.

Definition

A computation algorithm A provides a randomized additive (φ, δ)-approximation for a problem P with actual solution v^* if the value v returned by A is such that

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Hoeffding's Inequality

Let $X_1, \ldots, X_n$ be n independent random variables, each within the interval $[a, b]$, and $\bar{X} = \frac{1}{n} \sum_i X_i$ the empirical mean.

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Often too conservative!

Outline

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- This is completely independent of m!

Monte-Carlo Sampling

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Probabilistic Query Evaluation

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- Great, but these approximations may become more and more difficult to find as ε nears 0
- FPRAS (Fully Polynomial-time Randomized Approximation Scheme): PRAS whose overall complexity depends polynomially in $1/\varepsilon$

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- See `http://webcourse.cs.technion.ac.il/236605/Spring2015/ho/WCFiles/L9%20-%20QA%20in%20PDBs.pdf` for detailed explanations

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- But one can show there is no FPRAS for arbitary SQL queries

Technical tool: lineages

(slide from A. Amarilli)

The lineage of a query q on an instance I:

- Boolean function φ whose variables are the facts of I
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→ Lineage: $f_2 \wedge ((g_2 \wedge h_1) \vee (g_3 \wedge h_2)) = (f_2 \wedge g_2 \wedge h_1) \vee (f_2 \wedge g_3 \wedge h_2)$

Outline

Introduction

Intractable Classes

Deterministic Approximations

Randomized Approximations

Conclusion

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In brief

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- But additive approximations are generally not as good as multiplicative approximations
- Some problems have a FPRAS, which makes randomized approximation fully efficient and practical

To go further

- Claire Mathieu's MOOC on approximation algorithms
  - https://www.coursera.org/learn/approximation-algorithms-part-1
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