TD: Computational Geometry

Algorithmique et Programmation

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The goal of this exercise session is to review basic algorithms in computational geometry. We only consider 2-dimensional objects (2D points, segments, rays, polygons) for simplicity. We also ignore errors that may result of numerical approximations, and assume all arithmetic operations have constant cost.

1. Given two points $p_1 \neq (0,0)$ and $p_2 \neq (0,0)$, give a simple algorithm, not using any trigonometric function, to determine whether the quantity $(\alpha_2 - \alpha_1 \bmod 2\pi)$ (where α_1 and α_2 are the polar angles of p_1 and p_2) is 0, π, in $(0;\pi)$ or in $(\pi;2\pi)$.

2. Given three points p_1, p_2, and p_3 all distinct, give an algorithm to determine whether the succession of the segments p_1p_2 and p_2p_3 is a right turn, a left turn, a straight forward move, or a straight backward move.

3. Given four points p_1, p_2, p_3, p_4 all distinct, and using the previous questions, propose an algorithm to determine whether two segments p_1p_2 and p_3p_4 intersect. You can first assume that no three points are aligned, and then treat this special case.

4. Given a set S of n points, propose a $O(n^2 \log n)$ algorithm to determine whether there exist three aligned points in S.

5. Given an n-sided polygon P described by its successive vertices $p_1, p_2, \ldots, p_n$, propose a $O(n)$ algorithm to determine whether P is convex.

6. Given an n-sided convex polygon P and a point q, propose a $O(n)$ algorithm to determine whether q is the interior of P.

7. Given four points p_1, p_2, p_3, p_4 all distinct, propose an algorithm to determine whether the segment p_1p_2 and the half-line (ray) starting in p_3 and going through p_4 intersect.

8. Using the previous question, given an n-sided polygon P and a point q, propose a $O(n)$ algorithm to determine whether q is the interior of P. Make sure the algorithm works even in degenerate cases.

9. The *convex hull* of a set of n points S is the smallest convex polygon whose interior or boundary contains all points in S. Show that one of the point of the convex hull can be determined in $O(n)$.

10. Propose a $O(n \log n)$ algorithm to compute the convex hull of a set of n points. One can use the characterization of question 5 of convex polygons, keeping in mind the characterization of question 6 of points within convex polygons.

11. Propose a $O(nh)$ algorithm to compute the convex hull of a set of n points, where h is the size of the convex hull.

12. Give examples where each of the previous two algorithms is better than the other.