CS3236: Homework 9

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Due Monday, October 27, 2pm

Submission: Submission can be made in person before the beginning of the lecture, or by upload to the IVLE Workbin. You may submit handwritten answers (possibly scanned), but they must be **clearly readable**. Answers that are not readable may receive only partial points. Late answers are still accepted on Tuesday by 2pm (on the IVLE Workbin only), but receive only 50% of the possible points. Any later submissions receive no points. Assignments are marked out of 20 points.

1. (2 points) Do exercise 1.4 in the book.
2. (4 points) Find the distance in exercise 13.17 in the book.
3. (4 points) Consider the $[7,4]$-Hamming code. Suppose that we extend this code to a $[8,4]$-code by adding an overall parity check bit to each codeword. That is, the last bit corresponds to the parity of all 8 transmitted bits (parity bits included).
   a) (1 points) Write down the generator matrix G of this code in standard form.
   b) (3 points) Show that the code is self-dual.
4. (5 points) Consider a $[4,2]$-ternary code called the tetracode, i.e., a code over a ternary alphabet $\{0,1,2\}$ given by the generator matrix
$$G = \begin{pmatrix} 1 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 \end{pmatrix}.$$
   a) (2 points) Bring the generator matrix into standard form.
   b) (3 points) Show that the tetracode is self-dual.
5. (5 points) A punctured code $\mathcal{C}^*$ is a code $\mathcal{C}$ in which we removed the i-th coordinate of the codeword. Consider $i \in \{2,6\}$ and write down the resulting punctured codes for the two codes above. What can you say about the distance of the resulting code? What is the dual code and do the codes remain self-dual? *(In the ternary case, the distance between two words remains the number of places where they differ, as in the binary case. For example $d(012120,020120) = 2$. In particular, $d(0,1) = d(0,2) = d(1,2) = 1$.)*