CS3236: Homework 3

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Due Monday, September 1, 2pm

Submission: Submission can be made in person before the beginning of the lecture, or by upload to the IVLE Workbin. You may submit handwritten answers (possibly scanned), but they must be **clearly readable**. Answers that are not readable may receive only partial marks. Late answers are still accepted on Tuesday (on the IVLE Workbin only), but receive only 50% of the possible marks. Any later submissions receive no marks. Assignments are marked out of 20 points.

1. (4 points) Do exercise 4.16 in the book.
2. Consider an n-bit string $x \in \{0,1\}^n$ in which each bit $x_j \in \{0,1\}$ is chosen independently with probability $p(X_j = 0) = 1/5$ and $p(X_j = 1) = 4/5$.
   a) (3 points) How would you compress this string if $n = 3$? Describe one possible method of lossy compression (any reasonable method you can think of is fine). Analyze your choice of methods and determine (1) how well you compressed, i.e., how long the compressed strings are (2) what is the failure probability of your scheme.
   b) (3 points) For the same situation ($n = 3$), propose a method of lossless compression (again, any reasonable method is fine) and determine how long compressed strings are (in expectation).
   c) (4 points) We are back to lossy compression. What would you do if n is very large? Describe your method and how well it works.
   d) (2 points) Let's now assume that $p(X_j = 0) = p(X_j = 1) = 1/2$. How well does your lossy compression method perform now?
3. (4 points) Consider an n-bit string $x \in \{0,1\}^n$ in which the all-zero string is chosen with probability $P_X(x = 00\ldots0) = 1/2$ and with probability $1/2$ we choose one of the remaining strings uniformly at random. Sketch an idea on how we might perform lossless compression of such strings. How good is such a scheme with respect to Shannon entropy?
4. (Bonus) Do exercise 4.19 in the book.