CS3236: Homework 12

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Due Friday, November 14, 11:59pm

Submission: Submission can be made in person (during lecture or tutorial times), or by upload to the IVLE Workbin. You may submit handwritten answers (possibly scanned), but they must be **clearly readable**. Answers that are not readable may receive only partial points. Note that the due date is identical to the due date for the project. Assignments are marked out of 20 points.

1. a) (4 points) Exhibit a Turing machine that produces, given the empty input, the string 1^n for an arbitrary *nonnegative* integer n.

   b) (2 points) Deduce an upper bound on $C(1^n)$, both as an exact expression and as a $O(\cdot)$ expression (use the encoding of Turing machines from the lecture).

2. (4 points) Show that there exists a constant β such that for any binary sequence x,

$$C(xx) \leq C(x) + \beta.$$

3. (4 points) Give an example of families of binary sequences $(w_n)_{n\in\mathbb{N}}$ such that $C(w_n) = o(\log \ell(w_n))$ (that is, whose Kolmogorov complexity is asymptotically significantly lower than the logarithm of their size). You do not need to give a precise description of the corresponding family of Turing machine, but explain how they would be constructed and what their asymptotic size is.

4. (6 points) To be precise, we distinguish in this exercise a Turing machine T (which is an object formed of states and transitions) from the partial function $f_T : \{0,1\}^* \to \{0,1\}^*$ computed by T. A property P is a mapping from partial functions: $f : \{0,1\}^* \to \{0,1\}^*$ to Booleans; in other words, a property is something that is either true or false for any partial function: $f : \{0,1\}^* \to \{0,1\}^*$. One such property is for example the function f being constant, or being defined for a single value, or being monotone for some given order. A property of the function f_T computed by a Turing machine T is called *trivial* if it holds either for every function $f_{T'}$ computed by a Turing machine T' or for none. Show that for any non-trivial property P there exists no Turing machine R such that $R(E(T)) = 1$ if P is true for f_T, with T an arbitrary Turing machine, and $R(E(T)) = 0$ if P is false for f_T.