Provenance and Probabilities in Relational Databases

From Theory to Practice

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Provenance management

- Data management all about query evaluation
- What if we want something more than the query result?
    - Where does the result come from?
    - Why was this result obtained?
    - How was the result produced?
    - What is the probability of the result?
    - How many times was the result obtained?
    - How would the result change if part of the input data was missing?
    - What is the minimal security clearance I need to see the result?
    - What is the most economical way of obtaining the result?
    - How can a result be explained in layman terms?
- Provenance management: along with query evaluation, record additional bookkeeping information allowing to answer the questions above

Data model

- Relational data model: data decomposed into relations, with labeled attributes. . .

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name	position	city	classification
John	Director	New York	unclassified
Paul	Janitor	New York	restricted
Dave	Analyst	Paris	confidential
Ellen	Field agent	Berlin	secret
Magdalen	Double agent	Paris	top secret
Nancy	HR director	Paris	restricted
Susan	Analyst	Berlin	secret

Data model

- Relational data model: data decomposed into relations, with labeled attributes…
- … with an extra provenance annotation for each tuple (think of it first as a tuple id)

name	position	city	classification	prov
John	Director	New York	unclassified	t_1
Paul	Janitor	New York	restricted	t_2
Dave	Analyst	Paris	confidential	t_3
Ellen	Field agent	Berlin	secret	t_4
Magdalen	Double agent	Paris	top secret	t_5
Nancy	HR director	Paris	restricted	t_6
Susan	Analyst	Berlin	secret	t_7

Relations and databases

Formally:

- A relational schema $\mathcal{R}$ is a finite sequence of distinct attribute names; the arity of $\mathcal{R}$ is $|\mathcal{R}|$
- A database schema is a mapping from relation names to relational schemas, with finite support
- A tuple over relation schema $\mathcal{R}$ is a mapping from $\mathcal{R}$ to data values; each tuple comes with a provenance annotation
- A relation instance (or relation) over $\mathcal{R}$ is a finite set of tuples over $\mathcal{R}$
- A database instance (or database) over database schema $\mathcal{D}$ is a mapping from the support of $\mathcal{D}$ mapping each relation name R to a relation instance over $\mathcal{D}(R)$

Queries

- A query is an arbitrary function that maps databases over a fixed database schema $\mathcal{D}$ to relations over some relational schema $\mathcal{R}$
- The query does not consider or produce any provenance annotations; we will give semantics for the provenance annotations of the output, based on that of the input
- In practice, one often restricts to specific query languages:
  - Monadic-Second Order logic (MSO)
  - First-Order logic (FO) or the relational algebra
  - SQL with aggregate functions
  - etc.

Outline

Boolean provenance [Imieliński and Lipski, 1984]

- $\mathcal{X} = \{x_1, x_2, \dots, x_n\}$ finite set of Boolean events
- Provenance annotation: Boolean function over $\mathcal{X}$, i.e., a function of the form: $(\mathcal{X} \to \{\bot, \top\}) \to \{\bot, \top\}$
- Interpretation: possible-world semantics
  - every valuation $\nu : \mathcal{X} \to \{\bot, \top\}$ denotes a possible world of the database
  - the provenance of a tuple on ν evaluates to $\bot$ or $\top$ depending whether this tuple exists in that possible world
  - for example, if every tuple of a database is annotated with the indicator function of a distinct Boolean event, the set of possible worlds is the set of all subdatabases

Example of possible worlds

name	position	city	classification	prov
John	Director	New York	unclassified	t_1
Paul	Janitor	New York	restricted	t_2
Dave	Analyst	Paris	confidential	t_3
Ellen	Field agent	Berlin	secret	t_4
Magdalen	Double agent	Paris	top secret	t_5
Nancy	HR director	Paris	restricted	t_6
Susan	Analyst	Berlin	secret	t_7

$$\nu: \quad \begin{array}{ccccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 \\ \top & \top & \top & \top & \top & \top & \top \end{array}$$

Example of possible worlds

name	position	city	classification	prov
John	Director	New York	unclassified	t_1
Dave	Analyst	Paris	confidential	t_3
Magdalen	Double agent	Paris	top secret	t_5
Susan	Analyst	Berlin	secret	t_7

$$\nu: \quad \begin{array}{ccccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 \\ \top & \bot & \top & \bot & \top & \bot & \top \end{array}$$

Boolean provenance of query results

- $\nu(D)$: the subdatabase of D where all tuples whose provenance annotation evaluates to $\perp$ by ν is removed
- The Boolean provenance $\mathrm{prov}_{q,D}(t)$ of tuple $t \in q(D)$ is the function:

$$\nu \mapsto \begin{cases} \top \text{ if } t \in q(\nu(D)) \\ \perp \text{ otherwise} \end{cases}$$

Example (What cities are in the table?)

name	position	city	classification	prov
John	Director	New York	unclassified	t_1
Paul	Janitor	New York	restricted	t_2
Dave	Analyst	Paris	confidential	t_3
Ellen	Field agent	Berlin	secret	t_4
Magdalen	Double agent	Paris	top secret	t_5
Nancy	HR director	Paris	restricted	t_6
Susan	Analyst	Berlin	secret	t_7

city	prov
New York	$t_1 \vee t_2$
Paris	$t_3 \vee t_5 \vee t_6$
Berlin	$t_4 \vee t_7$

Application: Probabilistic databases
[Green and Tannen, 2006, Suciu et al., 2011]

- Tuple-independent database: each tuple t in a database is annotated with independent probability $\Pr(t)$ of existing
- Probability of a possible world $D' \subseteq D$:

$$\Pr(D') = \prod_{t \in D'} \Pr(t) \times \prod_{t \in D' \setminus D} (1 - \Pr(t'))$$

- Probability of a tuple for a query q over D:

$$\Pr(t \in q(D)) = \sum_{\substack{D' \subseteq D \\ t \in q(D')}} \Pr(D')$$

- If $\Pr(x_i) := \Pr(t_i)$ where x_i is the provenance annotation of tuple t_i then $\Pr(t \in q(D)) = \Pr(\mathrm{prov}_{q,D}(t))$
- Computing the probability of a query in probabilistic databases thus amounts to computing Boolean provenance, and then computing the probability of a Boolean function
- Also works for more complex probabilistic models

Example of probability computation

name	position	city	classification	prov	prob
John	Director	New York	unclassified	t_1	0.5
Paul	Janitor	New York	restricted	t_2	0.7
Dave	Analyst	Paris	confidential	t_3	0.3
Ellen	Field agent	Berlin	secret	t_4	0.2
Magdalen	Double agent	Paris	top secret	t_5	1.0
Nancy	HR director	Paris	restricted	t_6	0.8
Susan	Analyst	Berlin	secret	t_7	0.2

city	prov
New York	$t_1 \vee t_2$
Paris	$t_3 \vee t_5 \vee t_6$
Berlin	$t_4 \vee t_7$

Example of probability computation

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John	Director	New York	unclassified	t_1	0.5
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Ellen	Field agent	Berlin	secret	t_4	0.2
Magdalen	Double agent	Paris	top secret	t_5	1.0
Nancy	HR director	Paris	restricted	t_6	0.8
Susan	Analyst	Berlin	secret	t_7	0.2

city	prov	prob
New York	$t_1 \vee t_2$	$1-(1-0.5)\times(1-0.7)=0.85$
Paris	$t_3 \vee t_5 \vee t_6$	1.00
Berlin	$t_4 \vee t_7$	$1-(1-0.2)\times(1-0.2)=0.36$

What now?

- How to compute Boolean provenance for practical query languages? What complexity?
- Can we do more with provenance?
- How should we represent provenance annotations?
- How can we implement support for provenance management in a relational database management system?

Outline

Commutative semiring $(K, \mathbb{0}, \mathbb{1}, \oplus, \otimes)$

- Set K with distinguished elements $\mathbb{0}$, $\mathbb{1}$
- $\oplus$ associative, commutative operator, with identity $\mathbb{0}_K$:
  - $a \oplus (b \oplus c) = (a \oplus b) \oplus c$
  - $a \oplus b = b \oplus a$
  - $a \oplus \mathbb{0} = \mathbb{0} \oplus a = a$
- $\otimes$ associative, commutative operator, with identity $\mathbb{1}_K$:
  - $a \otimes (b \otimes c) = (a \otimes b) \otimes c$
  - $a \otimes b = b \otimes a$
  - $a \otimes \mathbb{1} = \mathbb{1} \otimes a = a$
- $\otimes$ distributes over $\oplus$:

$$a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)$$

- $\mathbb{0}$ is annihilating for $\otimes$:

$$a \otimes \mathbb{0} = \mathbb{0} \otimes a = \mathbb{0}$$

Example semirings

- $(\mathbb{N}, 0, 1, +, \times)$: counting semiring
- $(\{\bot, \top\}, \bot, \top, \vee, \wedge)$: Boolean semiring
- $(\{\mathit{unclassified}, \mathit{restricted}, \mathit{confidential}, \mathit{secret}, \mathit{top\ secret}\}, \mathit{top\ secret}, \mathit{unclassified}, \min, \max)$: security semiring
- $(\mathbb{N} \cup \{\infty\}, \infty, 0, \min, +)$: tropical semiring
- $(\{\text{Boolean functions over } \mathcal{X}\}, \bot, \top, \vee, \wedge)$: semiring of Boolean functions over $\mathcal{X}$
- $(\mathbb{N}[\mathcal{X}], 0, 1, +, \times)$: semiring of integer-valued polynomials with variables in $\mathcal{X}$ (also called How-semiring or universal semiring, see further)
- $(\mathcal{P}(\mathcal{P}(\mathcal{X})), \emptyset, \{\emptyset\}, \cup, \Cup)$: Why-semiring over $\mathcal{X}$ $(A \Cup B := \{a \cup b \mid a \in A, b \in B\})$

Semiring provenance [Green et al., 2007]

- We fix a semiring $(K, \mathbb{0}, \mathbb{1}, \oplus, \otimes)$
- We assume provenance annotations are in K
- We consider a query q from the positive relational algebra (selection, projection, renaming, cross product, union; joins can be simulated with renaming, cross product, selection, projection)
- We define a semantics for the provenance of a tuple $t \in q(D)$ inductively on the structure of q

Selection, renaming

Provenance annotations of selected tuples are unchanged

Example $(\rho_{\text{name}\to\text{n}}(\sigma_{\text{city}=\text{“New York”}}(R)))$

name	position	city	classification	prov
John	Director	New York	unclassified	t_1
Paul	Janitor	New York	restricted	t_2
Dave	Analyst	Paris	confidential	t_3
Ellen	Field agent	Berlin	secret	t_4
Magdalen	Double agent	Paris	top secret	t_5
Nancy	HR director	Paris	restricted	t_6
Susan	Analyst	Berlin	secret	t_7

n	position	city	classification	prov
John	Director	New York	unclassified	t_1
Paul	Janitor	New York	restricted	t_2

Projection

Provenance annotations of identical, merged, tuples are $\oplus$-ed

Example ($\pi_{\text{city}}(R)$)

name	position	city	classification	prov
John	Director	New York	unclassified	t_1
Paul	Janitor	New York	restricted	t_2
Dave	Analyst	Paris	confidential	t_3
Ellen	Field agent	Berlin	secret	t_4
Magdalen	Double agent	Paris	top secret	t_5
Nancy	HR director	Paris	restricted	t_6
Susan	Analyst	Berlin	secret	t_7

city	prov
New York	$t_1 \oplus t_2$
Paris	$t_3 \oplus t_5 \oplus t_6$
Berlin	$t_4 \oplus t_7$

Union

Provenance annotations of identical, merged, tuples are $\oplus$-ed

Example

$$\pi_{\text{city}}(\sigma_{\textit{ends-with}(\text{position},\text{“agent”})}(R)) \cup \pi_{\text{city}}(\sigma_{\text{position}=\text{“Analyst”}}(R))$$

name	position	city	classification	prov
John	Director	New York	unclassified	t_1
Paul	Janitor	New York	restricted	t_2
Dave	Analyst	Paris	confidential	t_3
Ellen	Field agent	Berlin	secret	t_4
Magdalen	Double agent	Paris	top secret	t_5
Nancy	HR director	Paris	restricted	t_6
Susan	Analyst	Berlin	secret	t_7

city	prov
Paris	$t_3 \oplus t_5$
Berlin	$t_4 \oplus t_7$

Cross product

Provenance annotations of combined tuples are $\otimes$-ed

Example

$$\pi_{\text{city}}(\sigma_{\textit{ends-with}(\text{position},\text{“agent”})}(R)) \bowtie \pi_{\text{city}}(\sigma_{\text{position}=\text{“Analyst”}}(R))$$

name	position	city	classification	prov
John	Director	New York	unclassified	t_1
Paul	Janitor	New York	restricted	t_2
Dave	Analyst	Paris	confidential	t_3
Ellen	Field agent	Berlin	secret	t_4
Magdalen	Double agent	Paris	top secret	t_5
Nancy	HR director	Paris	restricted	t_6
Susan	Analyst	Berlin	secret	t_7

city	prov
Paris	$t_3 \otimes t_5$
Berlin	$t_4 \otimes t_7$

What can we do with it?

counting semiring: count the number of times a tuple can be derived, multiset semantics

Boolean semiring: determines if a tuple exists when a subdatabase is selected

security semiring: determines the minimum clearance level required to get a tuple as a result

tropical semiring: minimum-weight way of deriving a tuple (think shortest path in a graph)

Boolean functions: Boolean provenance, as previously defined

integer polynomials: universal provenance, see further

Why-semiring: Why-provenance [Buneman et al., 2001], set of combinations of tuples needed for a tuple to exist

Example of security provenance

$$\pi_{\text{city}}(\sigma_{\text{name}<\text{name2}}(\pi_{\text{name,city}}(R) \bowtie \rho_{\text{name}\to\text{name2}}(\pi_{\text{name,city}}(R))))$$

name	position	city	prov
John	Director	New York	unclassified
Paul	Janitor	New York	restricted
Dave	Analyst	Paris	confidential
Ellen	Field agent	Berlin	secret
Magdalen	Double agent	Paris	top secret
Nancy	HR director	Paris	restricted
Susan	Analyst	Berlin	secret

city	prov
New York	restricted
Paris	confidential
Berlin	secret

Notes [Green et al., 2007]

- Computing provenance has a PTIME data complexity overhead
- Semiring homomorphisms commute with provenance computation: if there is a homomorphism from K to K', then one can compute the provenance in K, apply the homomorphism, and obtain the same result as when computing provenance in K'
- The integer polynomial semiring is universal: there is a unique homomorphism to any other commutative semiring that respects a given valuation of the variables
- This means all computations can be performed in the universal semiring, and homomorphisms applied next
- Two equivalent queries can have two different provenance annotations on the same database, in some semirings

Outline

Semirings with monus [Amer, 1984, Geerts and Poggi, 2010]

- Some semirings can be equipped with a $\ominus$ verifying:
  - $a \oplus (b \ominus a) = b \oplus (a \ominus b)$
  - $(a \ominus b) \ominus c = a \ominus (b + c)$
  - $a \ominus a = \mathbb{0} \ominus a = \mathbb{0}$
- Boolean function semiring with $\wedge\neg$, Why-semiring with $\setminus$, counting semiring with truncated difference...
- Most natural semirings (but not all semirings [Amarilli and Monet, 2016]!) can be extended into semirings with monus
- Sometimes strange things happen [Amsterdamer et al., 2011]: e.g, $\otimes$ does not always distribute over $\ominus$
- Allows supporting full relational algebra with the $\setminus$ operator, still PTIME
- Semantics for Boolean function semiring coincides with that of Boolean provenance

Difference

Provenance annotations of diff-ed tuples are $\ominus$-ed

Example

$$\pi_{\text{city}}(\sigma_{\textit{ends-with}(\text{position},\text{“agent”})}(R)) \setminus \pi_{\text{city}}(\sigma_{\text{position}=\text{“Analyst”}}(R))$$

name	position	city	classification	prov
John	Director	New York	unclassified	t_1
Paul	Janitor	New York	restricted	t_2
Dave	Analyst	Paris	confidential	t_3
Ellen	Field agent	Berlin	secret	t_4
Magdalen	Double agent	Paris	top secret	t_5
Nancy	HR director	Paris	restricted	t_6
Susan	Analyst	Berlin	secret	t_7

city	prov
Paris	$t_5 \ominus t_3$
Berlin	$t_4 \ominus t_7$

Provenance for aggregates

[Amsterdamer et al., 2011, Fink et al., 2012]

- Trickier to define provenance for queries with aggregation, even in the Boolean case
- One can construct a K-semimodule $K * M$ for each monoid aggregate M over a provenance database with a semiring in K
- Data values become elements of the semimodule

Example $(\mathrm{count}(\pi_{\mathrm{name}}(\sigma_{\mathrm{city}=\text{“Paris”}}(R)))$

$$t_3 * 1 + t_5 * 1 + t_6 * 1$$

Where-provenance [Buneman et al., 2001]

- Different form of provenance: captures from which database values come which output values
- Bipartite graph of provenance: two attribute values are connected if one can be produced from the other
- Axiomatized in [Buneman et al., 2001, Cheney et al., 2009]
- Cannot be captured by provenance semirings [Cheney et al., 2009], because of renaming (does not keep track of relation attributes), projection (does not remember which attribute values still exist), join (in a join, an output value comes from two different input values)

Outline

Provenance

Representation Systems for Provenance

Implementing Provenance Support

Conclusion

Representation systems

- In the Boolean semiring, the counting semiring, the security semiring: provenance annotations are elementary
- In the Boolean function semiring, the universal semiring, etc., provenance annotations can become quite complex
- Needs for compact representation of provenance annotations
- Lower the provenance computation complexity as much as possible

Provenance formulas

- Quite straightforward
- Formalism used in most of the provenance literature
- PTIME data complexity
- Expanding formulas (e.g., computing the monomials of a $\mathbb{N}[\mathcal{X}]$ provenance annotation) can result in an exponential blowup

Example

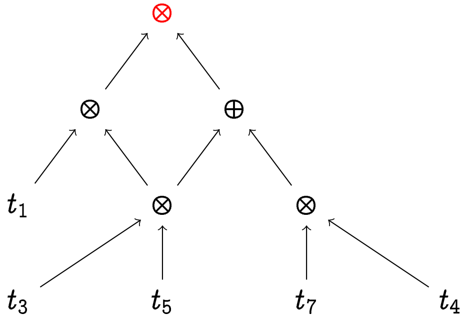
Is there a city with both an analyst and an agent, and if Paris is such a city, is there a director in the agency?

$$((t_3 \otimes t_5) \oplus (t_4 \otimes t_7)) \otimes ((t_3 \otimes t_5) \otimes t_1)$$

Provenance circuits [Deutch et al., 2014, Amarilli et al., 2015]

- Use arithmetic circuits (Boolean circuits for Boolean provenance) to represent provenance
- Every time an operation reuses a previously computed result, link to the previously created circuit gate
- Allow linear-time data complexity of provenance computation when restricted to bounded-treewidth databases [Amarilli et al., 2015] (MSO queries for Boolean provenance, positive relational algebra queries for arbitrary semirings)
- Formulas can be quadratically larger than provenance circuits for MSO formulas, $(\log\log)$-larger for positive relational algebra queries [Wegener, 1987, Amarilli et al., 2016]

Example provenance circuit



OBDD and d-DNNF

- Various subclasses of Boolean circuits commonly used:
  - OBDD: Ordered Binary Decision Diagrams
  - d-DNNF: deterministic Decomposable Negation Normal Form
- OBDDs can be obtained in PTIME data complexity on bounded-treewidth databases [Amarilli et al., 2016]
- d-DNNFs can be obtained in linear-time data complexity on bounded-treewidth databases
- Application: probabilistic query evaluation in linear-time data complexity on bounded-treewidth databases (d-DNNF evaluation is in linear-time)

Provenance cycluits [Amarilli et al., 2017]

- Cycluit (cyclic circuit): arithmetic circuit with cycles
- Well-defined semantics on some semirings where infinite loops do not matter
- Allows computing provenance in linear-time combined complexity for recursive queries of a certain form (ICG-Datalog of bounded body size [Amarilli et al., 2017], capturing α-acyclic conjunctive queries, 2RPQs, etc.), on bounded tree-width databases
- Related to provenance equation systems and formal series introduced in [Green et al., 2007]

Outline

Desiderata for a provenance-aware DBMS

- Extends a widely used database management system
- Easy to deploy
- Easy to use, transparent for the user
- Provenance automatically maintained as the user interacts with the database management system
- Provenance computation benefits from query optimization within the DBMS
- Allow probability computation based on provenance
- Any form of provenance can be computed: Boolean provenance, semiring provenance in any semiring (possibly, with monus), aggregate provenance, where-provenance, on demand

ProvSQL: Provenance within PostgreSQL (1/2)

[Senellart et al., 2018]

- Lightweight extension/plugin for PostgreSQL ≥ 9.5
- Provenance annotations stored as UUIDs, in an extra attribute of each provenance-aware relation
- A provenance circuit relating UUIDs of elementary provenance annotations and arithmetic gates stored as table
- All computations done in the universal semiring (more precisely, with monus, in the free semiring with monus; for where-provenance, in a free term algebra)

ProvSQL: Provenance within PostgreSQL (2/2)

[Senellart et al., 2018]

- Query rewriting to automatically compute output provenance attributes in terms of the query and input provenance attributes:
  - Duplicate elimination (DISTINCT, set union) results in aggregation of provenance values with $\oplus$
  - Cross products, joins results in combination of provenance values with $\otimes$
  - Difference rewritten in a join, with combination of provenance values with $\ominus$
- Additional circuit gates on projection, join for support of where-provenance
- Probability computation from the provenance circuits, via various methods (naive, sampling, compilation to d-DNNFs)

Challenges

- Low-level access to PostgreSQL data structures in extensions
- No simple query rewriting mechanism
- SQL is much less clean than the relational algebra
- Multiset semantics by default in SQL
- SQL is a very rich language, with many different ways of expressing the same thing
- Inherent limitations: e.g., no aggregation within recursive queries
- Implementing provenance computation should not slow down the computation
- User-defined functions, updates, etc.: unclear how provenance should work

ProvSQL: Current status

- Supported SQL language features:
  - Regular SELECT-FROM-WHERE queries (aka conjunctive queries with multiset semantics)
  - JOIN queries (regular joins and outer joins; semijoins and antijoins are not currently supported)
  - SELECT queries with nested SELECT subqueries in the FROM clause
  - GROUP BY queries (without aggregation)
  - SELECT DISTINCT queries (i.e., set semantics)
  - UNION's or UNION ALL's of SELECT queries
  - EXCEPT queries
- Longer term project: aggregate computation
- Try it (and see a demo) from `https://github.com/PierreSenellart/provsql`

Outline

Relational Data Provenance [Senellart, 2017]

- Quite rich foundations of provenance management:
  - Different types of provenance
  - Semiring formalism to unify most provenance forms
  - (Partial) extensions for difference, recursive queries, aggregation
  - Compact provenance representation formalisms
- Some theory still missing:
  - Provenance and updates
  - Going beyond the relational algebra for full semiring provenance
- Now is the time to work on concrete implementation
- Need good implementation to convince users they should track provenance!
- How to combine provenance computation and efficient query evaluation, e.g., through tree decompositions?

Merci.

https://github.com/PierreSenellart/provsql

https://youtu.be/iqzSNfGHbEE?vq=hd1080

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