

# On the Complexity of Managing Probabilistic XML Data

Pierre Senellart

Serge Abiteboul



*Principles Of Database Systems*, 13th June 2007

# Outline

- 1 Introduction
  - Motivation
  - Probabilistic Data Management
  - Complexity Issues
- 2 Prob-Trees
- 3 Equivalence of Prob-Trees
- 4 Prob-Trees with Additional Constraints
- 5 Conclusion

# Imprecise Data and Imprecise Tasks

## Observations

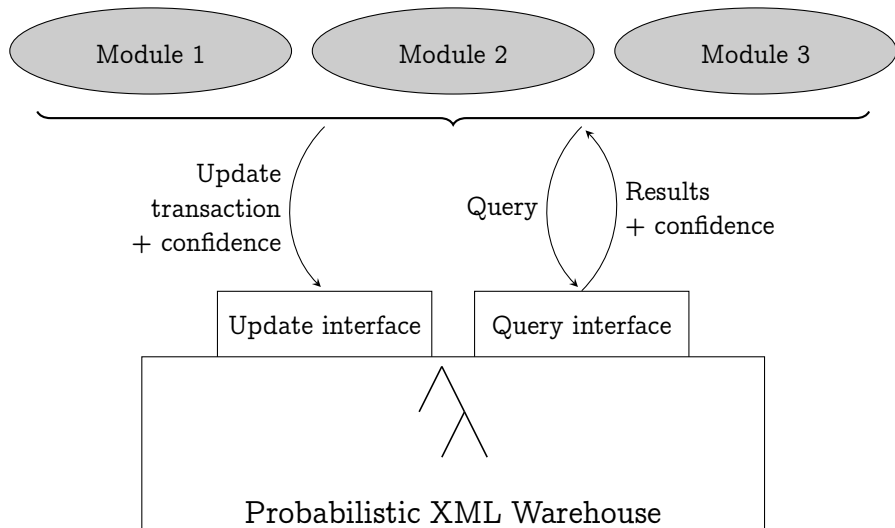
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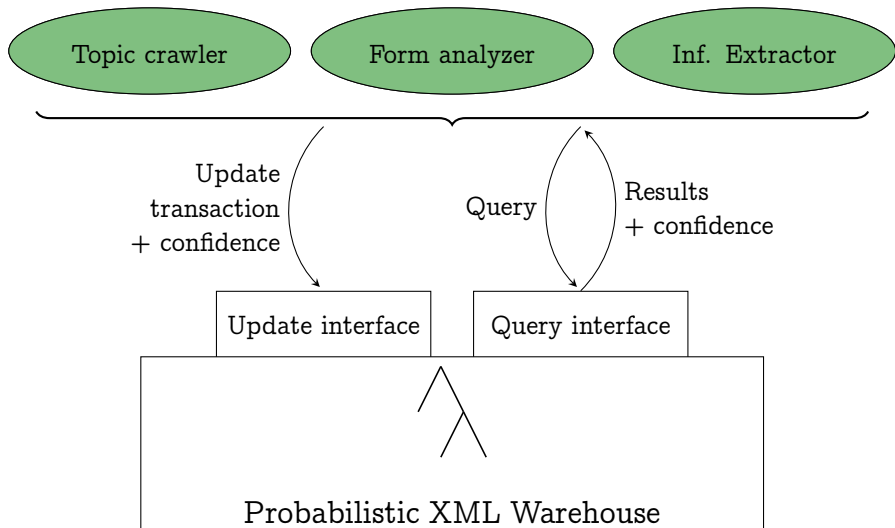
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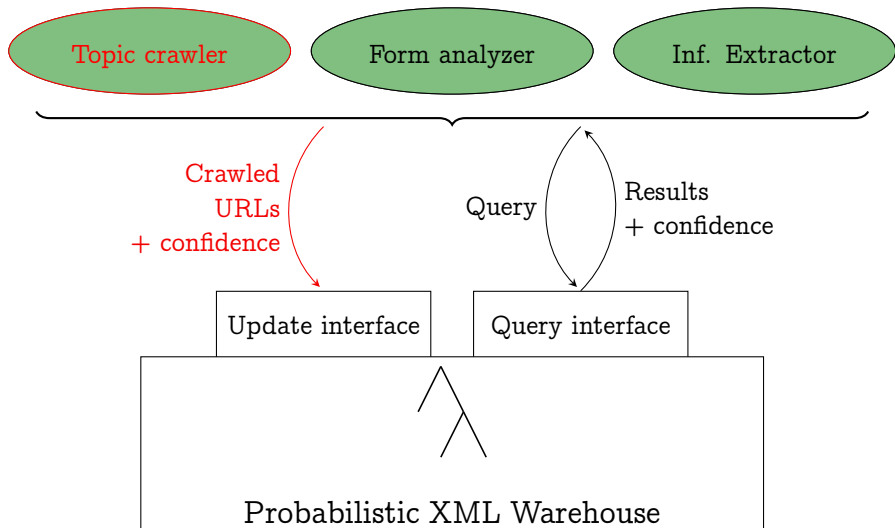
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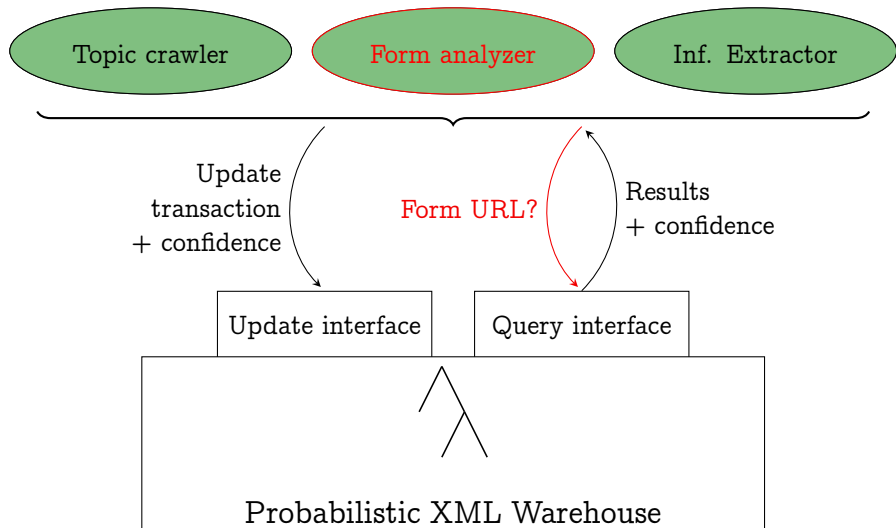
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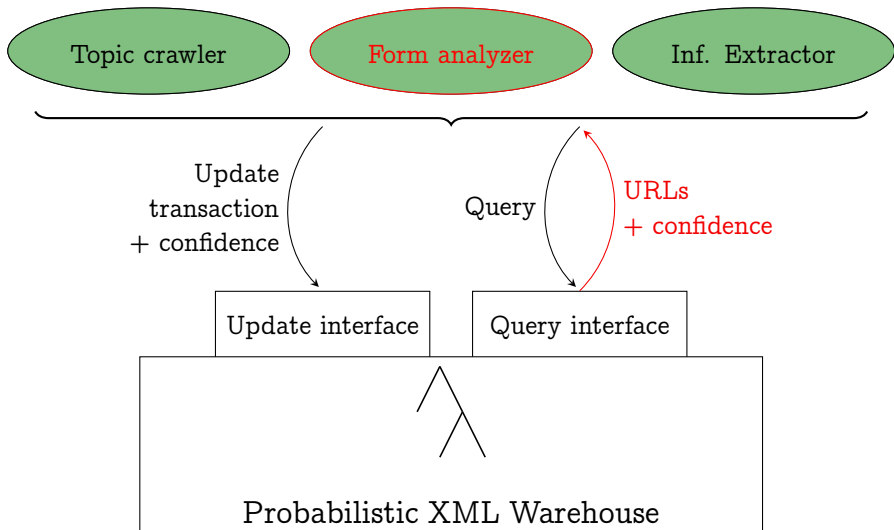


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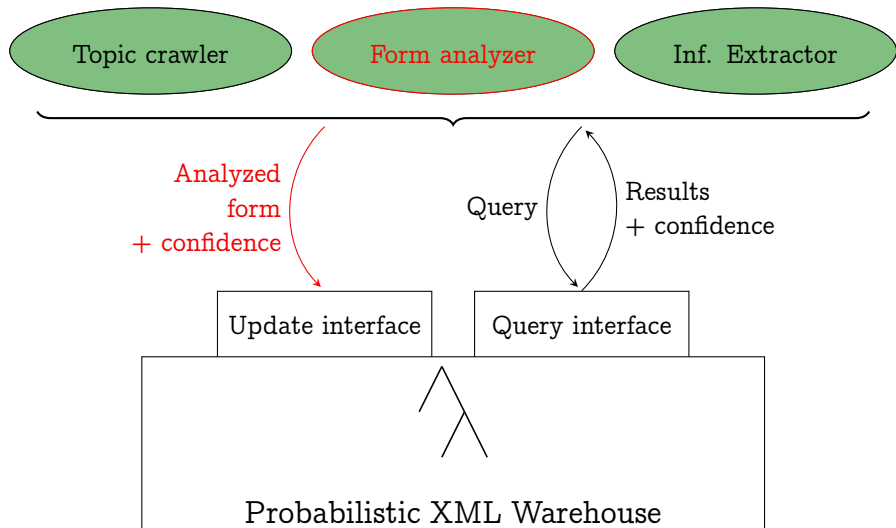




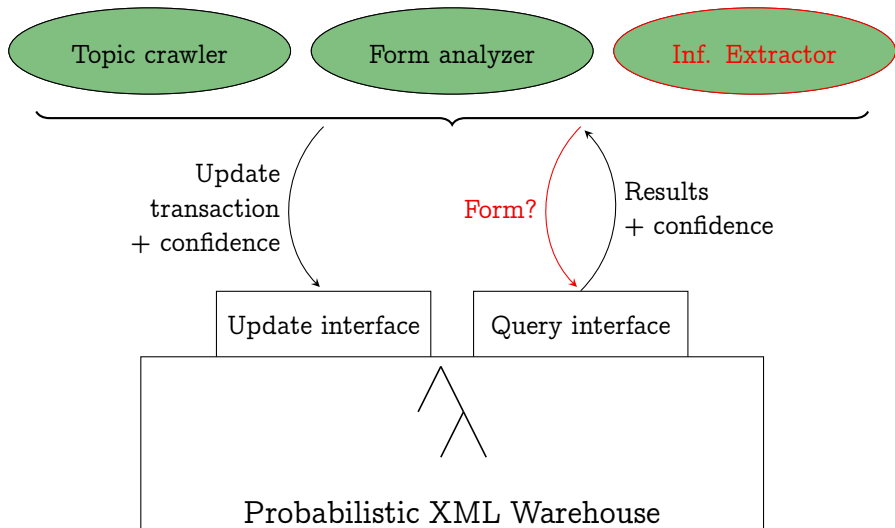
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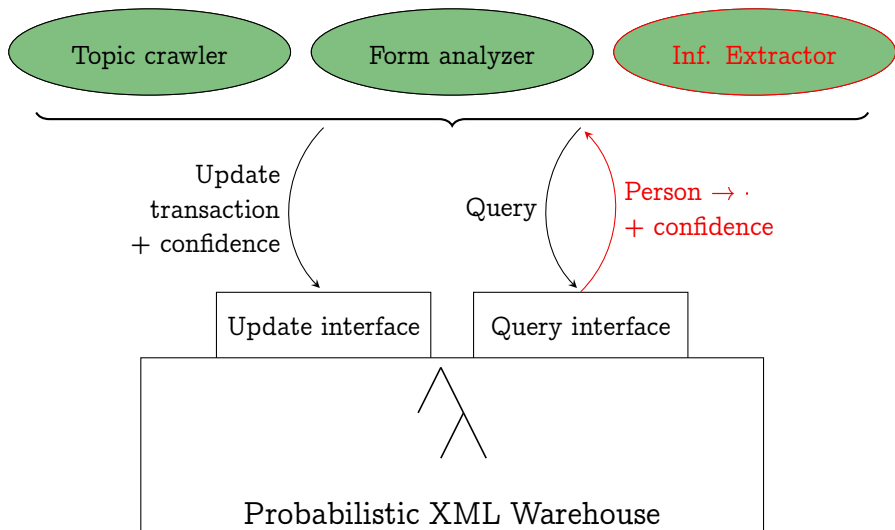
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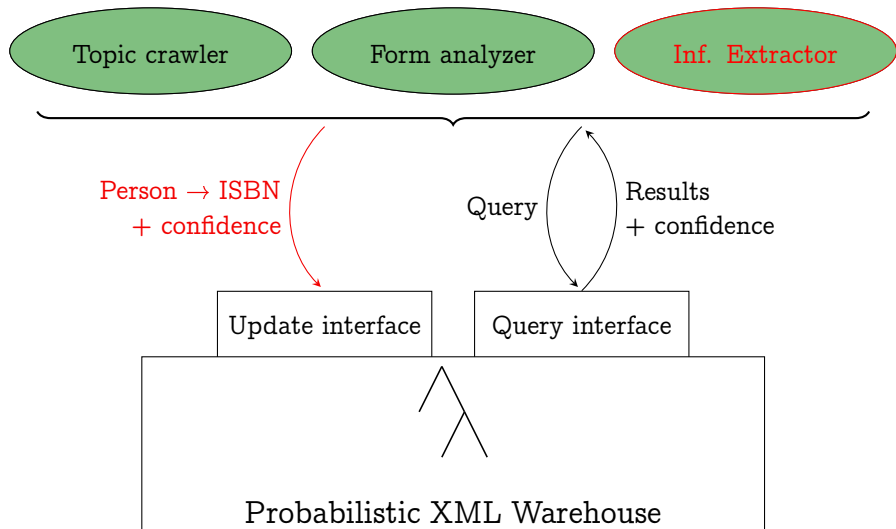
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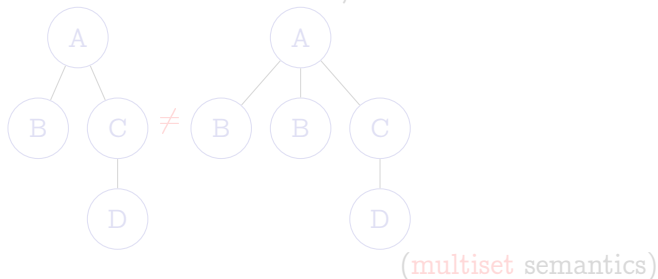
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# Probabilistic Trees

## Framework

- **Unordered** data trees
- Details: no attributes, no mixed content...



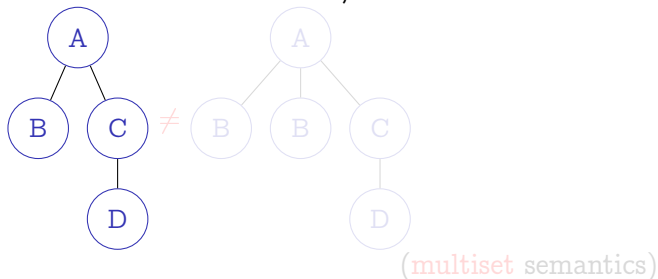
Sample space: Set of all such data trees.

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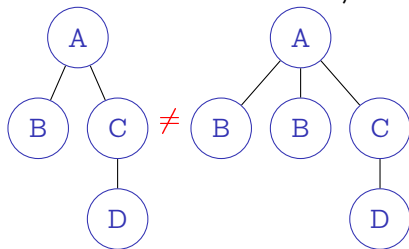
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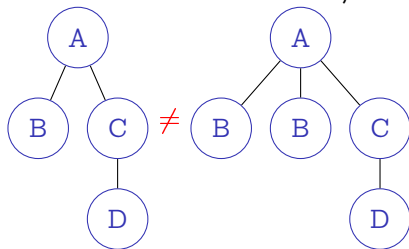
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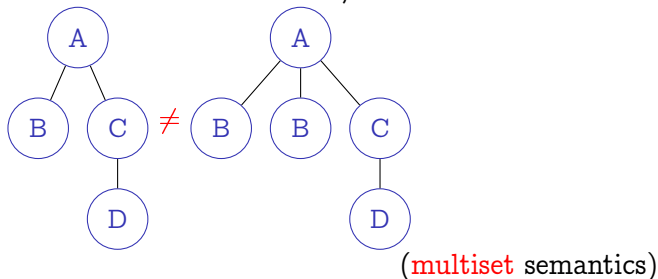
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# Complexity Issues

Prob-tree model defined in [Abiteboul & Senellart 2006]. Here, we tackle **complexity questions** about it:

- What is the complexity of **queries** and **updates**?
- Is this complexity **inherent** to the problem of managing tree-like probabilistic information?
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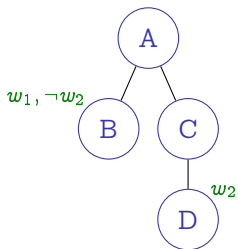
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  - The Prob-Tree Model
  - Queries and Updates
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# The Prob-Tree Model

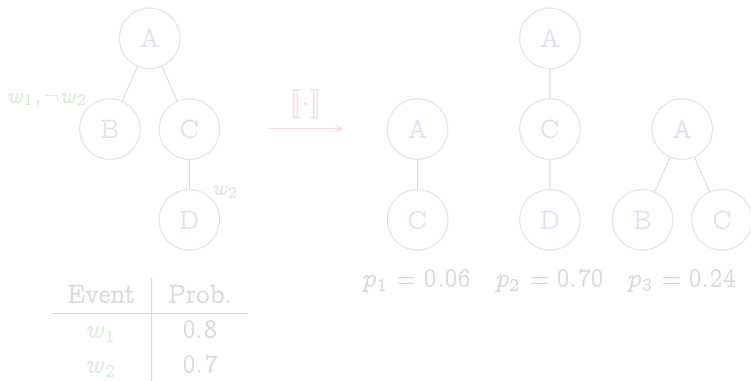
- Data tree with **event conditions** (conjunction of probabilistic events or negations of probabilistic events) **assigned to each node**.
- Probabilistic events are **boolean random variables**, assumed to be **independent**, with their own probability distribution.
- Representation *à la* [Imieliński & Lipksi 1984].



Event	Prob.
$w_1$	0.8
$w_2$	0.7

# Semantics of Prob-Trees

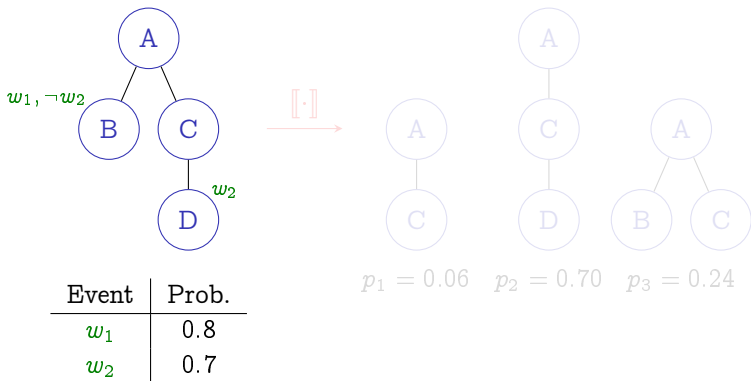
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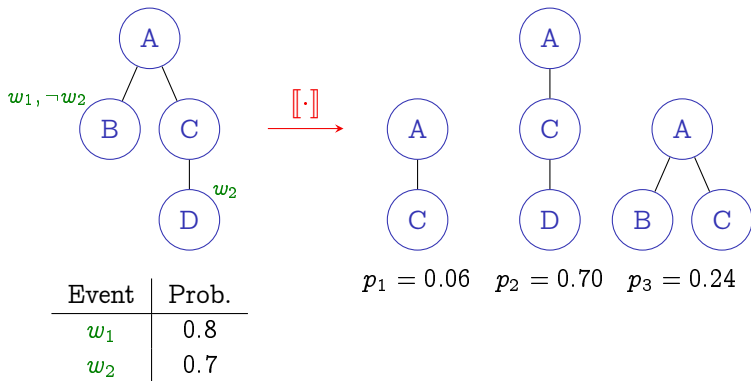
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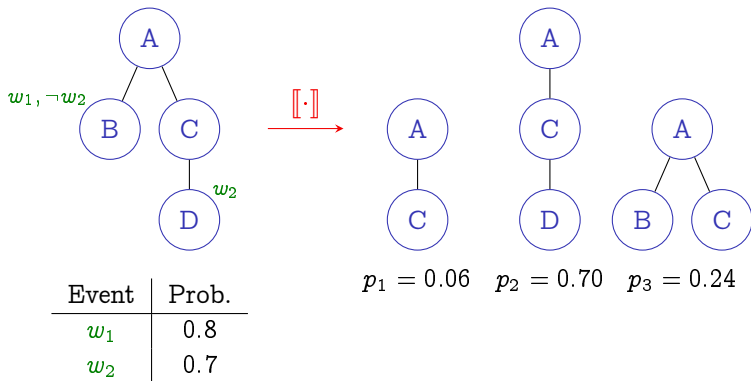
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# Locally Monotone Queries

**Query:** function that maps a data tree  $t$  to a **set of subtrees** of  $t$  containing its root.

## Definition

A query  $Q$  is **locally monotone** if, for any data trees  $u$ ,  $t'$  and  $t$  such that  $u \leq t' \leq t$ ,  $u \in Q(t) \iff u \in Q(t')$ .

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- "Return the root node if it has no A child, nothing otherwise." is **not** locally monotone.

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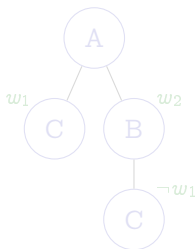
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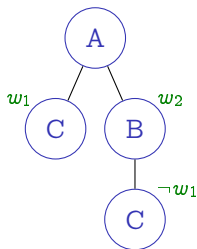
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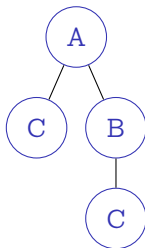
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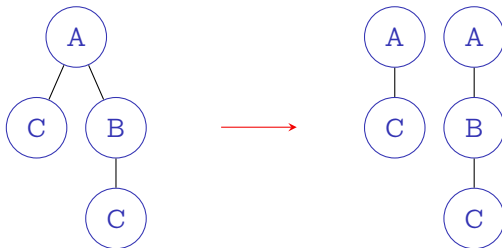


Underlying data tree.

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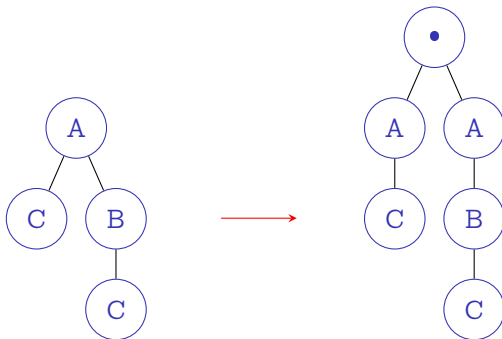
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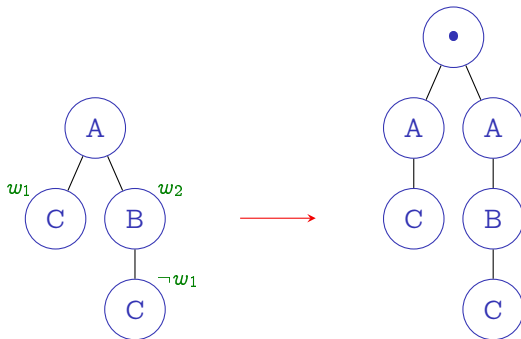
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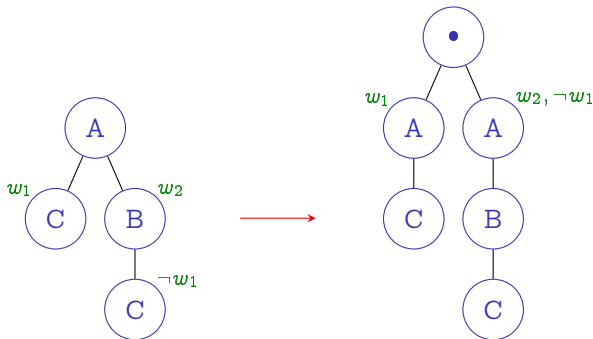
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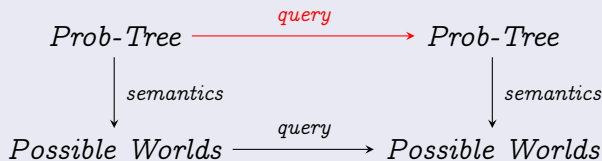
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# Consistence of Queries on Prob-Trees

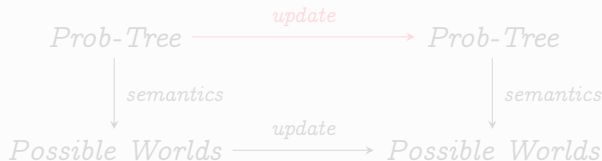
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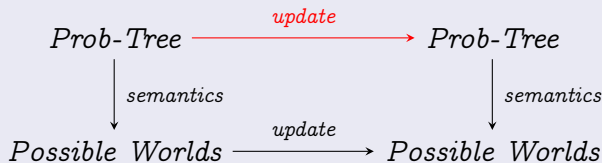
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$T$ : prob-tree with underlying data tree  $t$ .

$\text{time}(Q(t))$ : complexity of the query  $Q$  over the data tree  $t$ .

Upper bounds for operations on  $T$ :

Operation	Complexity
Query	$\text{time}(Q(t)) + \text{polynomial}$ in the size of $T$ , $Q(t)$
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  - Two Notions of Equivalence
  - Structural Equivalence
  - Semantic Equivalence
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# Two Notions of Equivalence

What does it mean for two prob-trees to **represent the same information**?

Two different notions:

**Structural Equivalence:** we keep the **same event variables**.

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Complexity results? Relation between these two notions?

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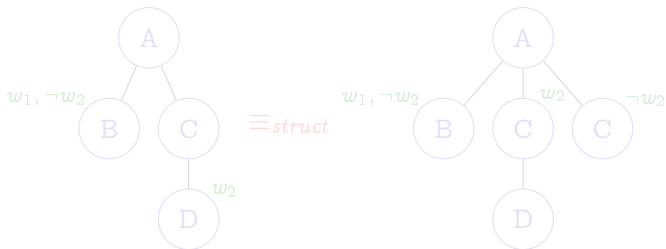
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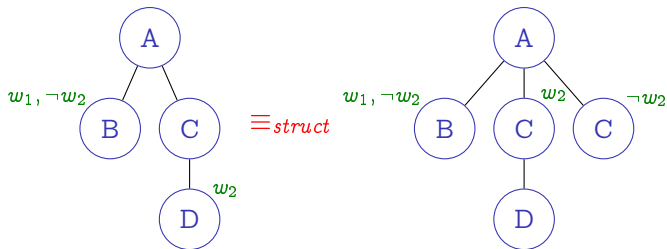




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# Complexity of Structural Equivalence

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*Structural Equivalence is a **coRP problem**: there exists a randomized polynomial-time algorithm that returns true if two prob-trees are equivalent, and false with probability  $\geq 1/2$  otherwise.*

Based on the notion of **count-equivalence**:

## Definition

Two propositional formulas  $\psi, \psi'$  in DNF are **count-equivalent** ( $\psi \stackrel{\pm}{\equiv} \psi'$ ) if, for every valuation of the variables of  $\psi$  and  $\psi'$ , the same number of disjuncts of  $\psi$  and  $\psi'$  are satisfied.

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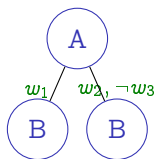
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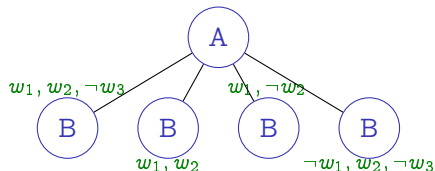
In a very simple case:



$$\iff w_1 \vee (w_2 \wedge \neg w_3)$$

$$\iff X_1 + X_2(1 - X_3)$$

$\equiv_{struct}$



$\stackrel{+}{\equiv}$

$$(w_1 \wedge w_2 \wedge \neg w_3) \vee (w_1 \wedge w_2) \vee$$

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$=$

$$X_1 X_2 (1 - X_3) + X_1 X_2 +$$

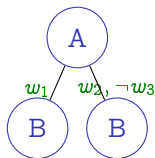
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(see [Green, Karvounarakis & Tannen 2007]).

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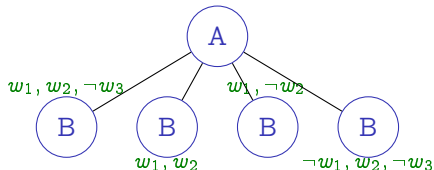
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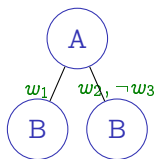
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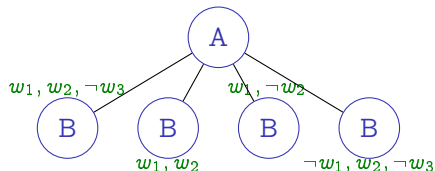
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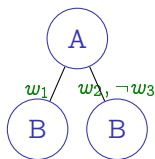
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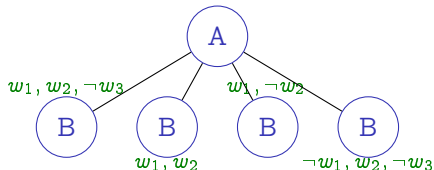
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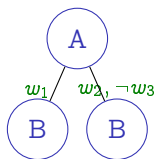
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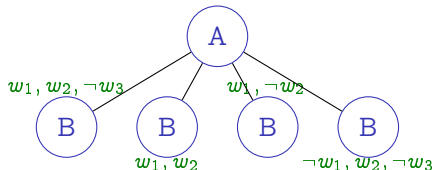
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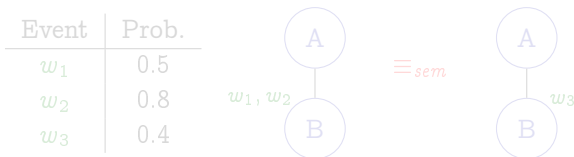
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Two prob-trees  $T$  and  $T'$  are **semantically equivalent** ( $T \equiv_{sem} T'$ ) if  $\llbracket T \rrbracket = \llbracket T' \rrbracket$ .

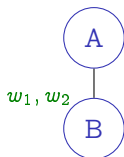
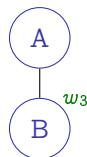


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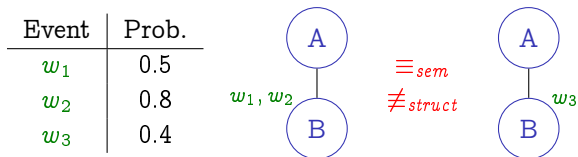
Event	Prob.
$w_1$	0.5
$w_2$	0.8
$w_3$	0.4


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## Facts

- 1 If  $T \equiv_{struct} T'$ , then  $T \equiv_{sem} T'$
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- 2 Prob-Trees
- 3 Equivalence of Prob-Trees
- 4 Prob-Trees with Additional Constraints**
  - Restriction to a Probability Threshold
  - DTD Validation
- 5 Conclusion



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- Is it possible to remove from a prob-tree **least probable** worlds?
- $\llbracket T \rrbracket_{\geq p}$  : set of possible worlds in  $\llbracket T \rrbracket$  whose probabilities are **greater than  $p$** .

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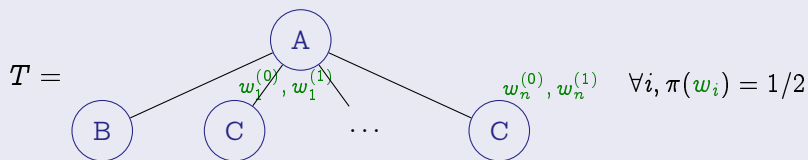
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Merci.

# Proof of the Exponential Complexity of Deletion

Proof.

**Deletion  $d$ :** “If the root has a C-child, then delete all B-children of the root.”



Then, it can be shown that if  $T' \equiv_{struct} d(T)$ , at least  $2^n$  literals appear in  $T'$ .



◀ Return to theorem

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