# On the Complexity of Deriving Schema Mappings from Database Instances

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# Different sources organize the same data differently

| | | |
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Querying websites using compact skeletons - all 11 versions »
A Rajaraman, **JD Ullman** - Journal of Computer and System Sciences, 2003 - Elsevier
Several commercial applications, such as online comparison shopping and process automation, require integrating information that is scattered across multiple w ebsites or XML documents. Much research has been devoted to this problem, **...**
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**[BOOK]** Wprowadzenie do teorii automatów, jezyków i obliczen
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Improving the efficiency of database-system teaching - all 3 versions »
**JD Ullman** - Proceedings of the 2003 ACM SIGMOD international conference …, 2003 - portal.acm.org
ABSTRACT The education industry has a very poor record of produc- tivity gains. In this brief article, I outline some of the w ays the teaching of a college course in database systems could be made more ecient, and sta time used **...**
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A survey of new directions in database systems. Ullman, JD Stanford University; This paper appears in: Database Systems for Advanced Applications, 2003. (DASFAA 2003). Proceedings. Eighth International **...**
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# Motivation

## Context

- Multiple data sources containing information about similar entities, with some redundancy (e.g., sources of the deep Web).
- Several different ways to present this information, i.e., several different schemata.
- No a priori information about (some of) these schemata.

How to know the relationships between these schemata, by just looking at the instances?

Other way to see this problem: Match operator on schema mappings, in the setting of data exchange.

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# Problem definition

## Problem

Given two (relational) database instances $I$ and $J$ with different schemata, what is the optimal description $\Sigma$ of $J$ knowing $I$ (with $\Sigma$ a finite set of formulas in some logical language)?

What does optimal implies:

- Conciseness of description.
- Validity of facts predicted by $I$ and $\Sigma$.
- All facts of $J$ explained by $I$ and $\Sigma$.

(Note the asymmetry between $I$ and $J$; context of data exchange where $J$ is computed from $I$ and $\Sigma$).

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# Outline

# Source-to-target tuple-generating dependencies

## Definition (Source-to-target tgd)

First-order formula of the form:

$$\forall \mathbf{x}\, \varphi(x) \rightarrow \exists \mathbf{y}\, \psi(x, y)$$

with:

- $\varphi$ conjunction of source relation atoms;
- $\psi$ conjunction of target relation atoms;
- all variables of $\mathbf{x}$ bound in $\varphi$.

## Example

$$\forall x_1 \forall x_2\, R_1(x_1, x_2) \wedge R_2(x_2) \rightarrow \exists y\, R'(x_1, y)$$

# Particular tgds

We consider two ways of having simpler tgds:

- Disallow existential quantifiers on the right hand-side: full tgds.
- Disallow cycles on both left- and right-hand sides: acyclic tgds. (Classical notion of acyclicity on hypergraphs extending the basic notion of acyclicity on graphs.)

**Examples**

$\forall x_1 \forall x_2 \forall x_3\ R_1(x_1, x_2) \wedge R_2(x_2, x_3) \wedge R_3(x_3, x_1) \rightarrow R'(x_1)$ is cyclic (and full).
$\forall x_1 \forall x_2 \forall x_3\ R_1(x_1, x_2) \wedge R_2(x_2, x_3) \rightarrow R'(x_1)$ is acyclic (and full).

We then consider the languages:

$\mathcal{L}_{\text{tgd}}$: arbitrary source-to-target tgds;
$\mathcal{L}_{\text{full}}$: full tgds;
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# How to define the pertinence of a set of tgds?

## Example

| $R$ |
|---|
| a |
| b |
| c |
| d |

| $R'$ | |
|---|---|
| a | a |
| b | b |
| c | a |
| d | d |
| g | h |

$\Sigma_0 = \varnothing$

$\Sigma_1 = \{\forall x\ R(x) \rightarrow R'(x,x)\}$

$\Sigma_2 = \{\forall x\ R(x) \rightarrow \exists y\ R'(x,y)\}$

$\Sigma_3 = \{\forall x_1 \forall x_2\ R(x_1) \wedge R(x_2) \rightarrow R'(x_1,x_2)\}$

$\Sigma_4 = \{\exists y_1 \exists y_2\ R'(y_1,y_2)\}$

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# Idea

- Size of a formula: number of occurrences of variables and constants.
- Cost of a schema mapping $\Sigma$: Size of the minimum repair of $\Sigma$ that is valid and explains all facts of $J$.
- Types of repairs considered:
  - "fix" a universal quantifier by adding conditions ($x = a$ or $x \neq a$);
  - "fix" an existential quantifier by giving corresponding constants ($\tau(\mathbf{x}) \rightarrow y = a$ with $\tau$ a conjunction of conditions on universally quantified variables);
  - add ground facts to the target instance.
- The problem is then to find a schema mapping of minimal cost.

# Example of cost computation

## Example

| $R$ |
|---|
| a |
| b |
| c |
| d |

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|---|---|
| a | a |
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$$\forall x \ R(x) \rightarrow R'(x, x)$$

Predicted $R'$

| | |
|---|---|
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## Example

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| a | a |
| b | b |
| c | a |
| d | d |
| g | h |

$$\forall x \; R(x) \land x \neq c \rightarrow R'(x, x)$$

Predicted $R'$

| | |
|---|---|
| a | a |
| b | b |
| d | d |

# Example of cost computation

## Example

| $R$ |
|---|
| a |
| b |
| c |
| d |

| $R'$ | |
|---|---|
| a | a |
| b | b |
| c | a |
| d | d |
| g | h |

$$\forall x\ R(x) \land x \neq c \rightarrow R'(x,x)$$

$$R'(c,a)$$

Predicted $R'$

| | |
|---|---|
| a | a |
| b | b |
| c | a |
| d | d |

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## Example

| $R$ |
|---|
| a |
| b |
| c |
| d |

| $R'$ | |
|---|---|
| a | a |
| b | b |
| c | a |
| d | d |
| g | h |

$$\forall x \; R(x) \land x \neq c \rightarrow R'(x, x)$$

$$R'(c, a)$$

$$R'(g, h)$$

Predicted $R'$

| | |
|---|---|
| a | a |
| b | b |
| c | c |
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# Example of cost computation

## Example

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$$\forall x\ R(x) \wedge x \neq c \rightarrow R'(x,x)$$

$$\exists y_1 \exists y_2\ R'(y_1,y_2) \wedge y_1 = c \wedge y_2 = a$$

$$\exists y_1 \exists y_2\ R'(y_1,y_2) \wedge y_1 = g \wedge y_2 = h$$

Predicted $R'$

| | |
|---|---|
| a | a |
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Cost: 17

Predicted $R'$

| | |
|---|---|
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# Problems considered

Decision problems of interest:

Cost : Is the cost of a given schema mapping less than $K$?

Optimality : Is a given schema mapping optimal?

Complexity? Algorithms?

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Complexity? Algorithms?

# Outline

# Behavior for simple operators

Consider the elementary operators of the relational algebra:

- Projection
- Intersection
- Selection (conjunction of atomic conditions)
- Cross Product
- Join (on a given attribute)

**Theorem**

*For any elementary operator $\gamma$, the tgd naturally associated with $\gamma$ is optimal with respect to $(I, \gamma(I))$ (or $(\gamma(J), J)$), under some basic assumptions.*

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# Examples of naturally associated tgds

## Examples

| | Condition | $I$ and $J$ | Optimal tgd |
|---|---|---|---|
| Projection | $I \neq \varnothing$ | $J = \pi_1(I)$ | $R(x,y) \to R'(x)$ |
| | $\pi_1(J) \cap \pi_2(J) = \varnothing$, $\|\pi_1(J)\| \geqslant 2$ | $I = \pi_1(J)$ | $R(x) \to \exists y\ R'(x,y)$ |
| Selection | $\|\sigma_\varphi(I)\| \geqslant \frac{\mathsf{size}(\varphi)+2}{3}$ | $J = \sigma_\varphi(I)$ | $R(x) \to R'(x)$ |
| | $\sigma_\varphi(J) \neq \varnothing$ | $I = \sigma_\varphi(J)$ | $R(x) \to R'(x)$ |
| Product | $R_1^I \neq \varnothing$, $R_2^I \neq \varnothing$ | $J = R_1^I \times R_2^I$ | $R_1(x) \wedge R_2(y) \to R'(x,y)$ |
| | ${R'_1}^J \neq \varnothing$, ${R'_2}^J \neq \varnothing$ | $I = {R'_1}^J \times {R'_2}^J$ | $R(x,y) \to R'_1(x) \wedge R'_2(y)$ |

# The Polynomial Hierarchy

| | |
|---|---|
| **P** | polynomial deterministic algorithm |
| **NP** | polynomial non-deterministic algorithm |
| **coNP** | complement **NP** |
| $\Sigma_2^P$ | polynomial non-deterministic with $\Sigma_1^P$ oracle |
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Union of all these classes: $\mathbf{PH} \subseteq \mathbf{PSPACE}$, the polynomial hierarchy.

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# (Combined) Complexity Results

| | $\mathcal{L}_{\text{tgd}}$ | $\mathcal{L}_{\text{full}}$ |
|---|---|---|
| Cost | $\Sigma_3^P$, $\Pi_2^P$-hard | $\Sigma_2^P$, (co)NP-hard |
| Optimality | $\Pi_4^P$, (co)NP-hard | $\Pi_3^P$, (co)NP-hard |

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# Vertex-Cover in $r$-partite $r$-uniform hypergraph

Vertex-Cover: find a set of vertices of minimal size that cover all (hyper)edges in a (hyper)graph.

- NP-complete for general (hyper)graphs.
- PTIME for bipartite graphs (Kőnig's theorem).

**Lemma**

*Vertex-Cover is NP-complete for $r$-partite $r$-uniform hypergraphs for $r \geqslant 3$.*

$r$-partite: partition of the set of vertices into $r$ sets, with no hyperedge spanning two vertices of the same set.

$r$-uniform: every hyperedge spans $r$ vertices.

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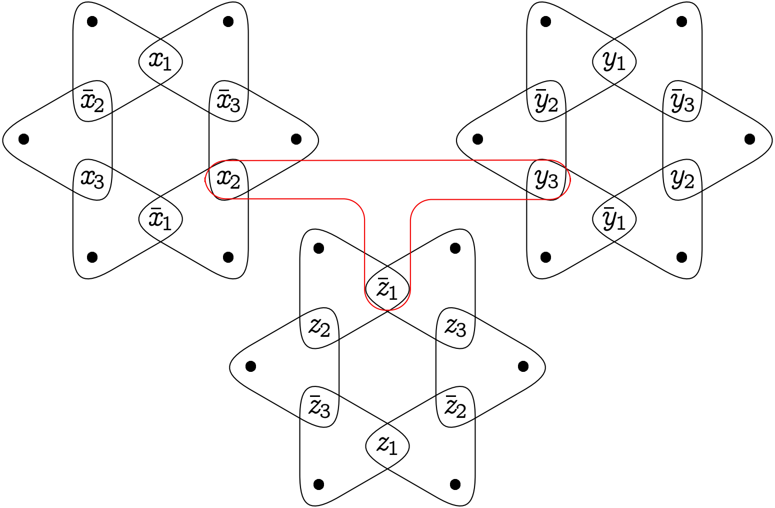
# Encoding of 3-SAT




$\neg z \vee x \vee y$

# Cost is NP-hard for $\mathcal{L}_{\text{facyc}}$

Reduction from Vertex-Cover in 3-partite 3-uniform hypergraphs.

Without $x = a$ repairs on the left-hand side of a tgd:

- $R(x_1, x_2, x_3) \rightarrow R'(x_1)$
- Source instance: hypergraph
- Target instance: empty

Cost: size of the tgd plus twice the minimum size of a vertex cover.

With $x = a$ repairs: a little more difficult, but feasible!

# Cost is NP-hard for $\mathcal{L}_{\text{facyc}}$

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# Outline

# Extension to Relational Calculus

- Definition of repairs can be extended to relational calculus.
- Same definition of cost, optimality.
- Cost is not recursive (but co-r.e.).
- Computability of Optimality: open (!).

# Other Cost Functions

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. . . because it can be exponential in the size of the schema mapping!

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- Link with Inductive Logic Programming?
- Heuristics?
- Approximation algorithms?
- Generalization of acyclicity?

Merci.