Tree decompositions for probabilistic data management

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(joint work with Antoine Amarilli, Pierre Bourhis, Silviu Maniu, Mikaël Monet)

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Slides due in large part to Antoine Amarilli

Query evaluation

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  - Example: R (arity 1), S (arity 2), T (arity 1)

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- Fragment $\mathcal{Q}$ of Boolean constant-free queries
  - Example: Boolean conjunctive queries (= existentially quantified conjunction of atoms)
  - Example of CQ: $q : \exists x\, y\ R(x) \wedge S(x, y) \wedge T(y)$

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  - Example of CQ: $q : \exists x\, y\ R(x) \wedge S(x, y) \wedge T(y)$
- Query evaluation problem for $\mathcal{Q}$ and $\mathcal{I}$:
  - Fix a query $q \in \mathcal{Q}$
  - Given an input instance $I \in \mathcal{I}$
  - Determine whether I satisfies q (written $I \models q$)
  - Data complexity: Complexity as a function of I, not q
  - Combined complexity: Complexity as a function of both I and q

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c	b	w	b

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- Semantics: (I, π) gives a probability distribution on $I' \subseteq I$:
  - Each fact $F \in I$ is either present or absent with probability $\pi(F)$
  - Facts are independent

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S		
a	a	1
b	v	.5
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- Probabilistic XML: (Cohen et al., 2009)
  - $\mathcal{Q}$ are deterministic tree automata, $\mathcal{I}$ are trees
  - PQE is PTIME in combined complexity

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- Does this extend to probabilistic QE?

Main result (Amarilli et al., 2015, 2016)

An instance-based dichotomy result on data complexity:

Upper bound. For $\mathcal{I}$ the treelike instances and $\mathcal{Q}$ the MSO queries

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Lower bound. For any unbounded-tw family $\mathcal{I}$ and $\mathcal{Q}$ FO queries

- PQE is #P-hard under RP reductions assuming
  - Signature arity is 2 (graphs)
  - High-tw instances in $\mathcal{I}$ are easily constructible

Outline

Technical tool: provenance

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General roadmap

- Use provenance for probabilistic query evaluation:
  - Compute a provenance representation efficiently
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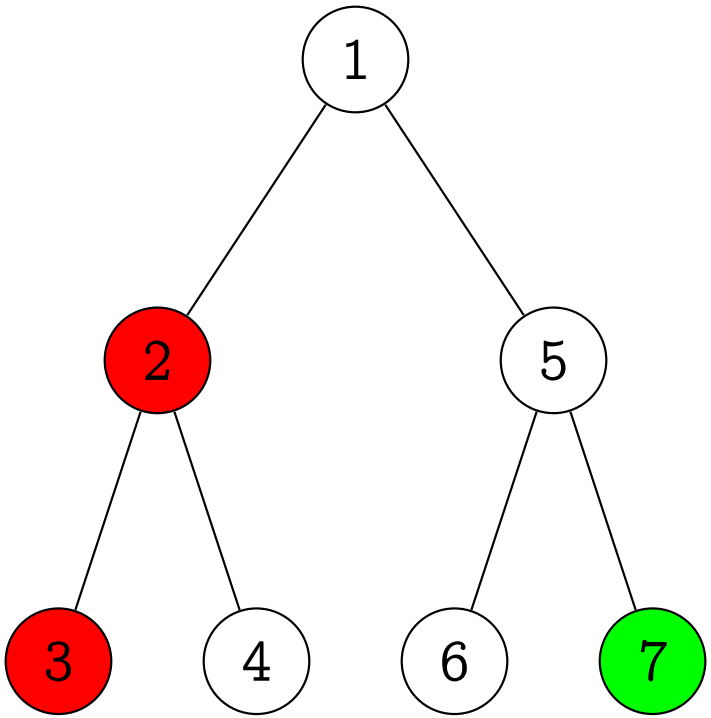
    (show it is not #P-hard as in the general case)

General roadmap

- Use provenance for probabilistic query evaluation:
  - Compute a provenance representation efficiently
  - Probability of the provenance = probability of the query
  - Compute the provenance probability efficiently
    (show it is not #P-hard as in the general case)
- To solve the PQE problem on treelike instances for MSO
  - First solve the problem on trees with tree automata
  - Then use the results of (Courcelle, 1990)

Uncertain trees



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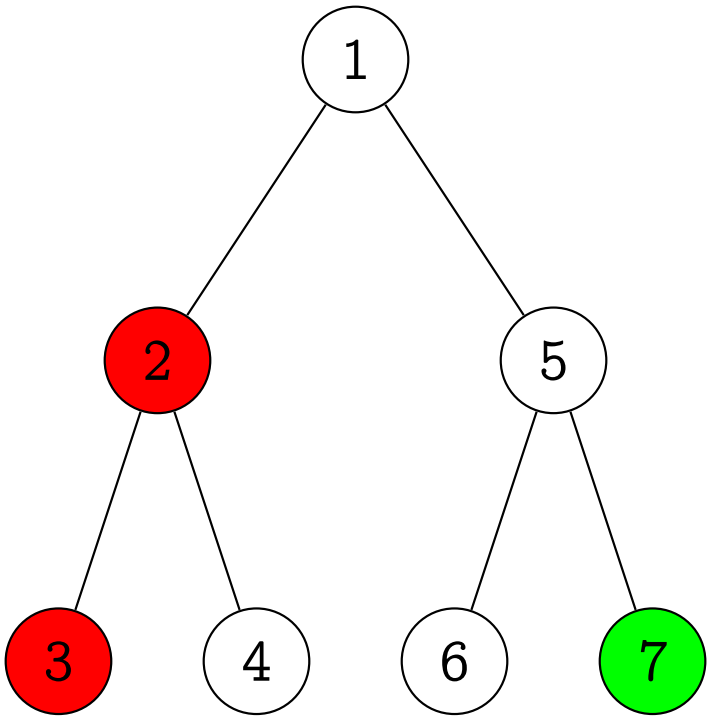

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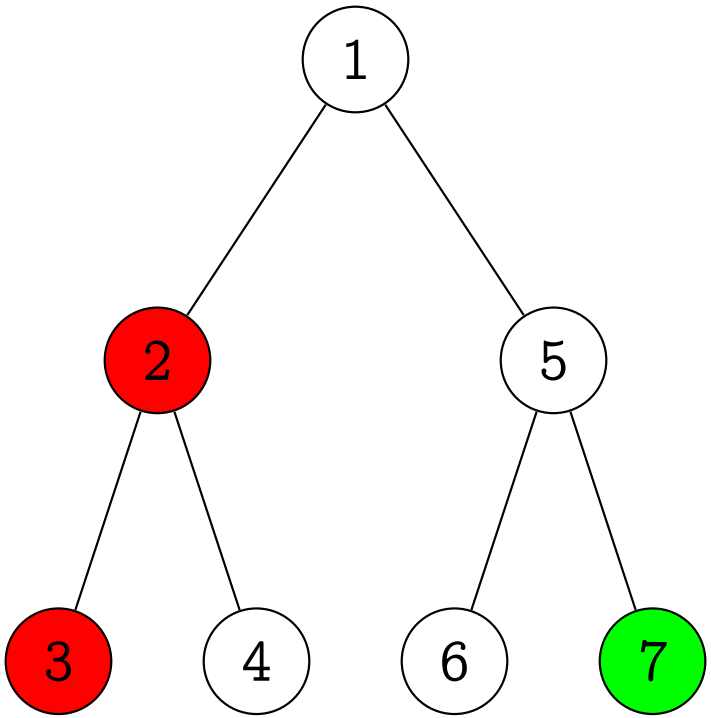

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"Is there both a red and a green node?"

Valuation: $\{2, 3, 7\}$

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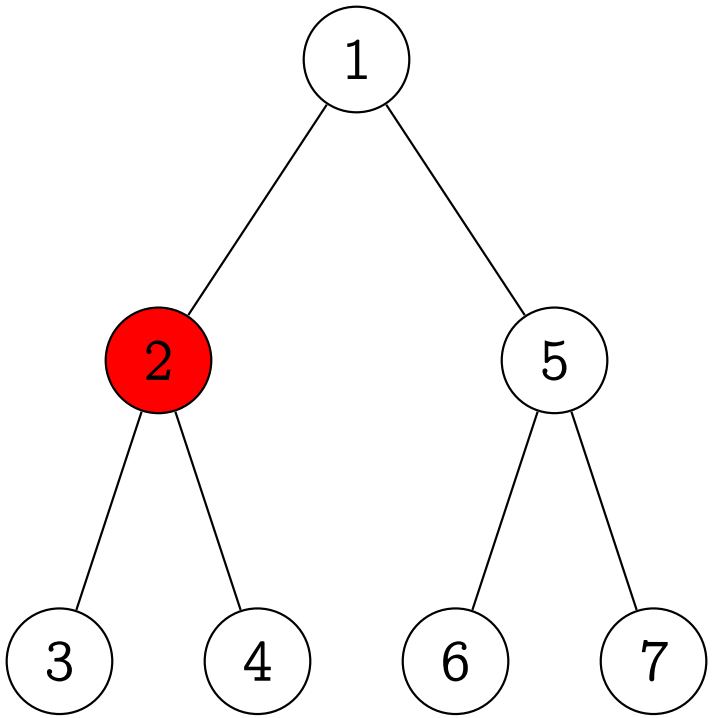

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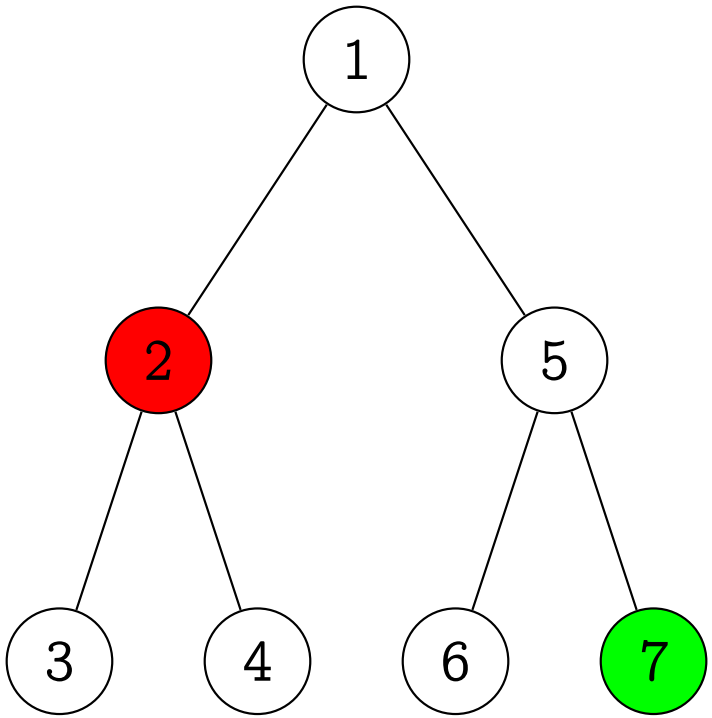

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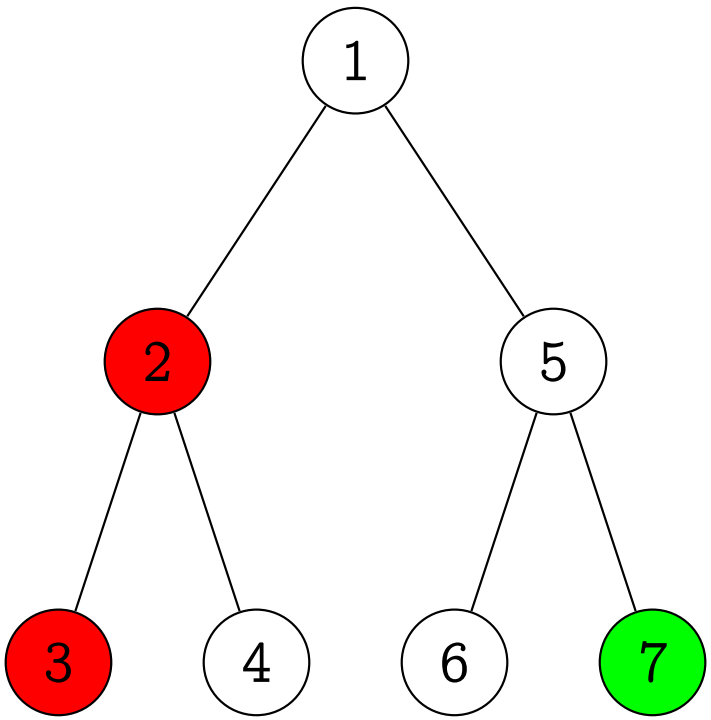

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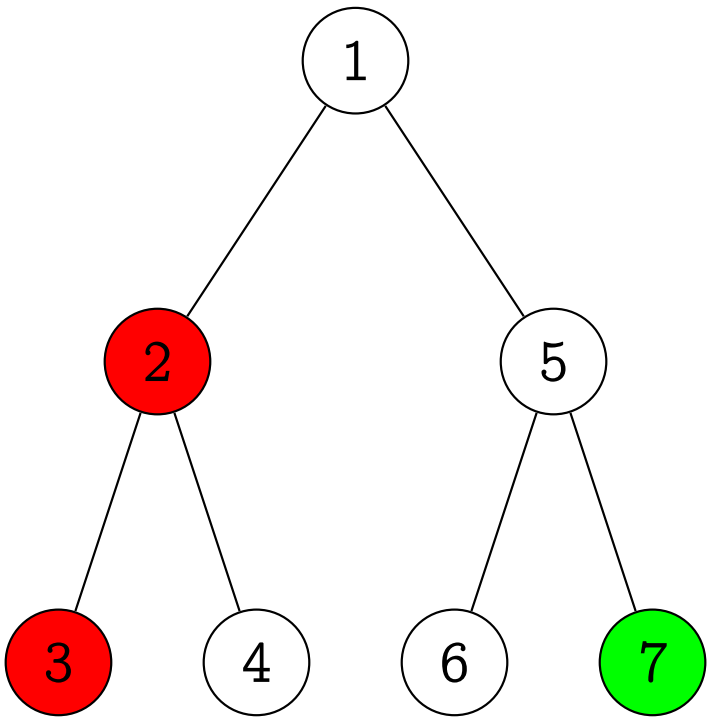

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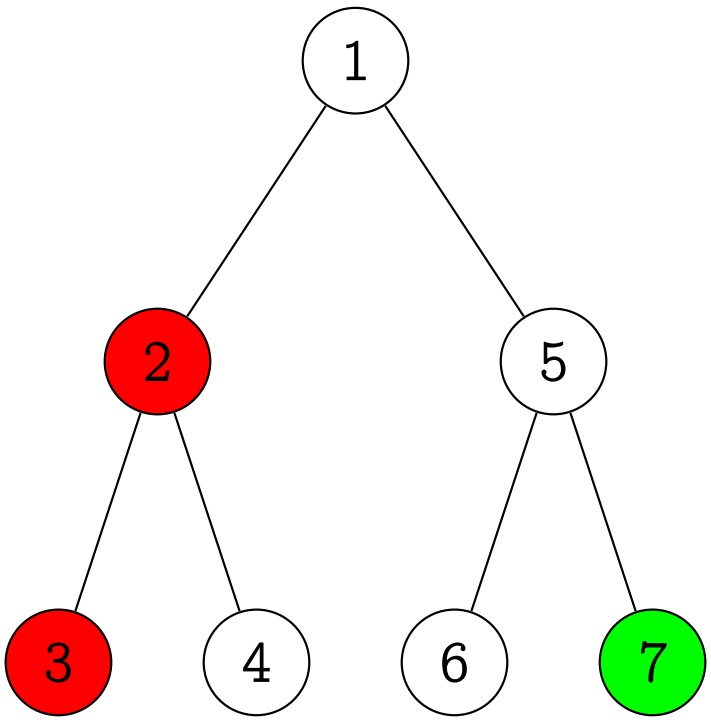

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- Provenance circuit of A on T (Deutch et al., 2014)
  - Boolean circuit C
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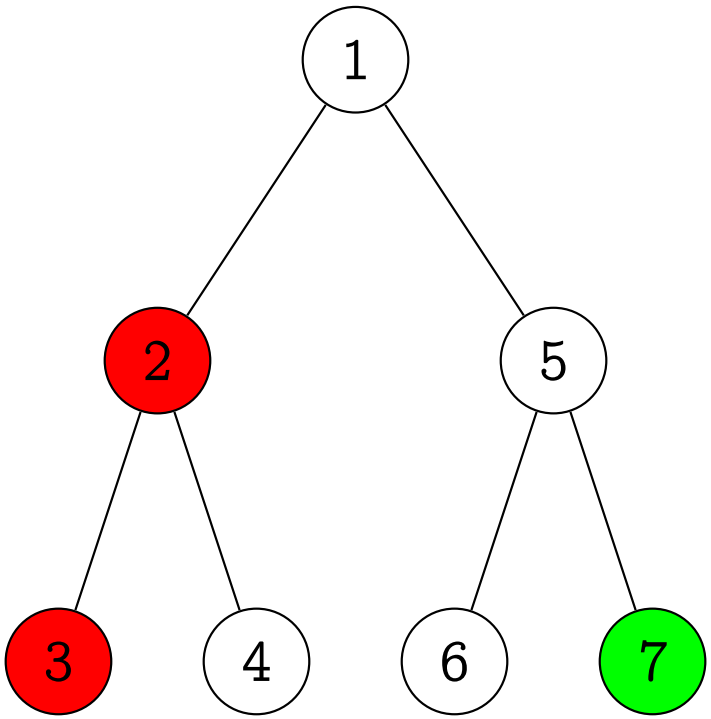

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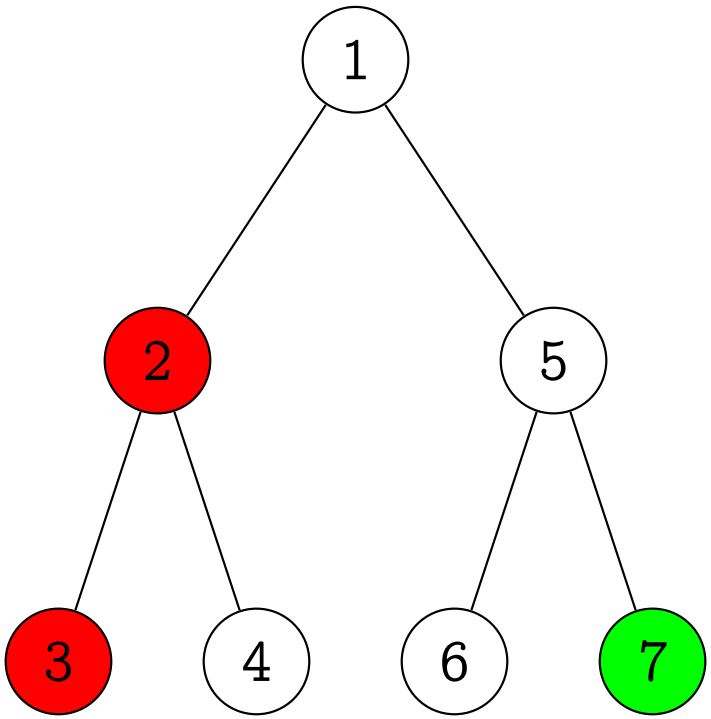

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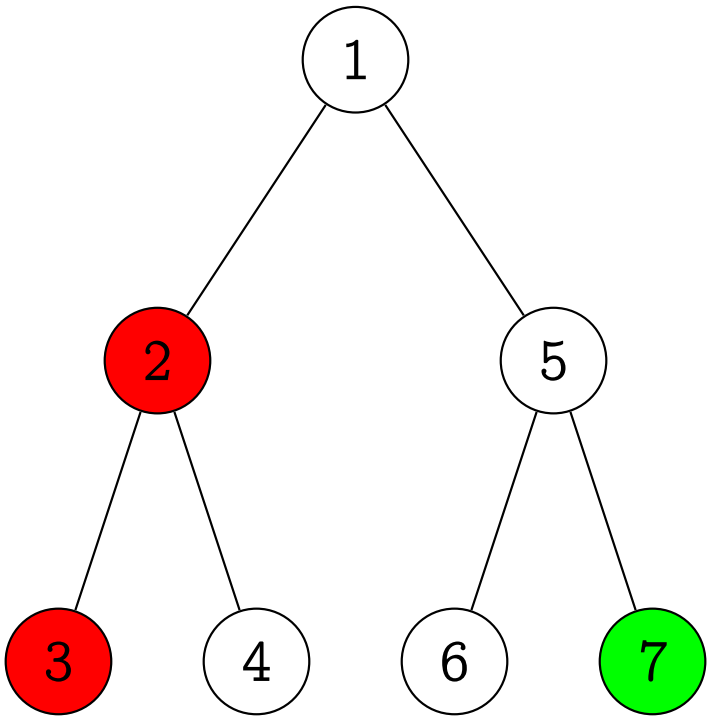

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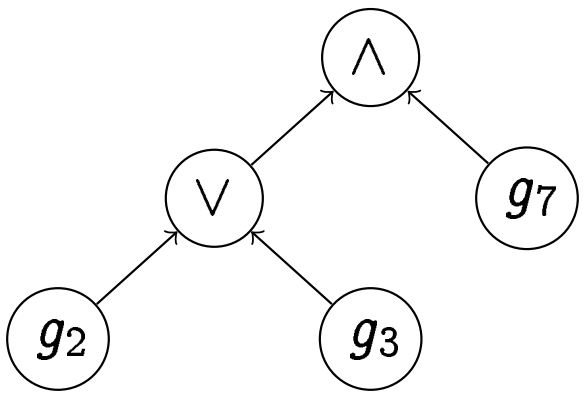

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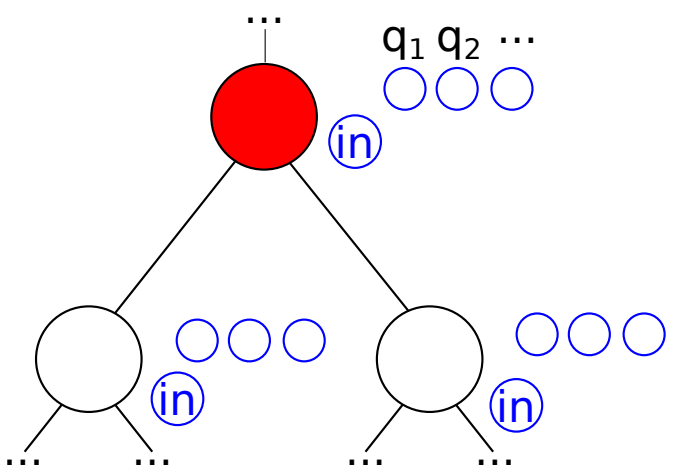

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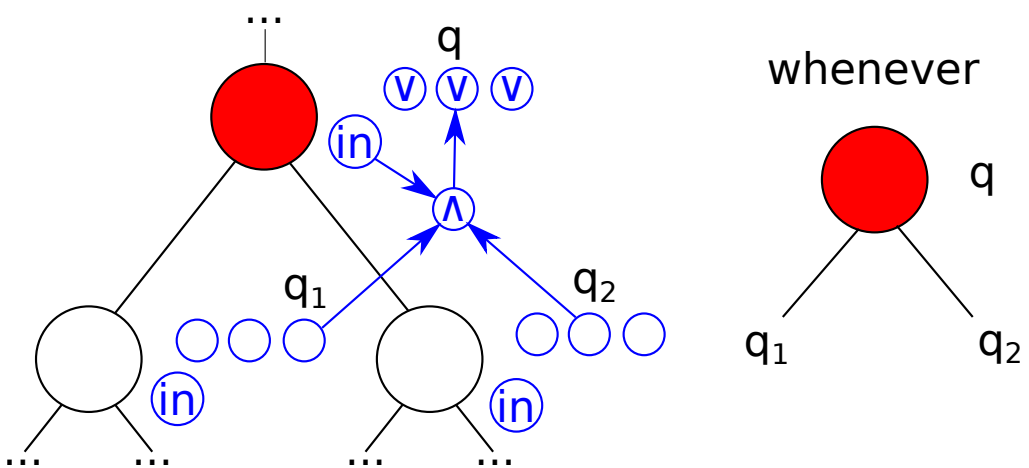

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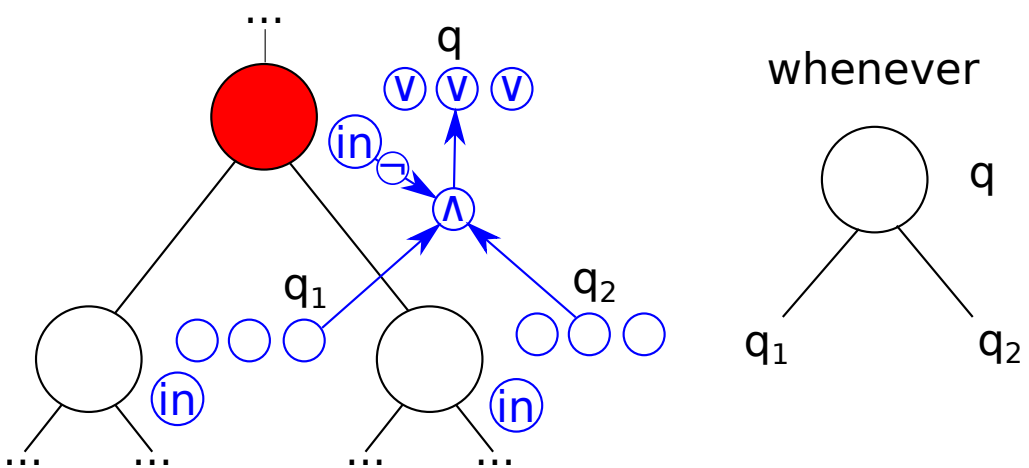

Treelike instances

- Treelike instance I
- Tree encoding: tree E on fixed alphabet, represents I
- MSO query on I translates to
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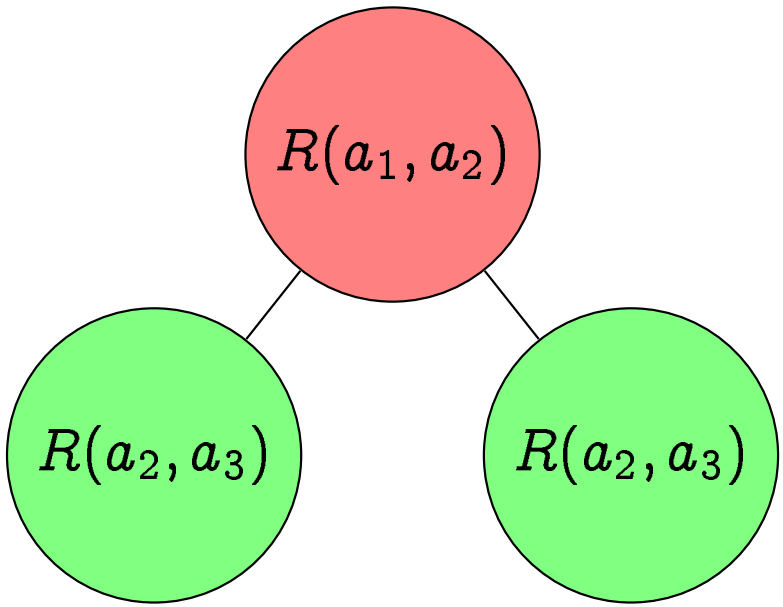

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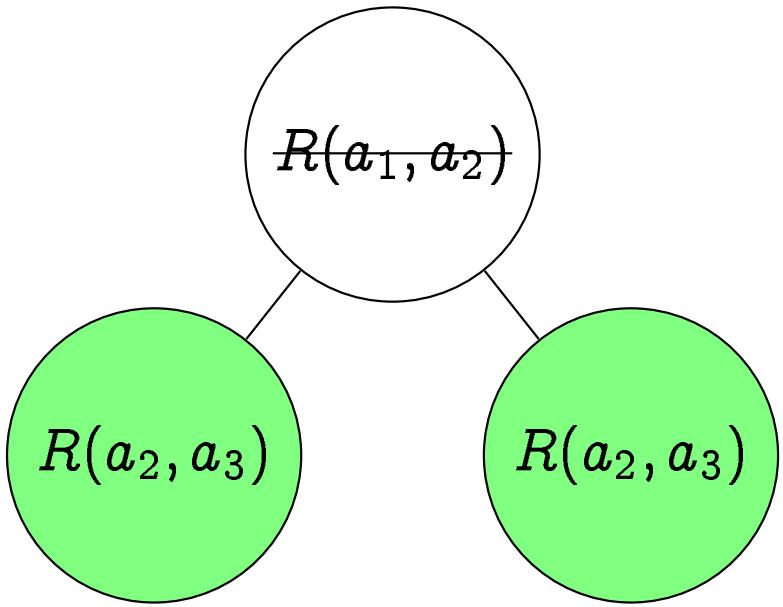

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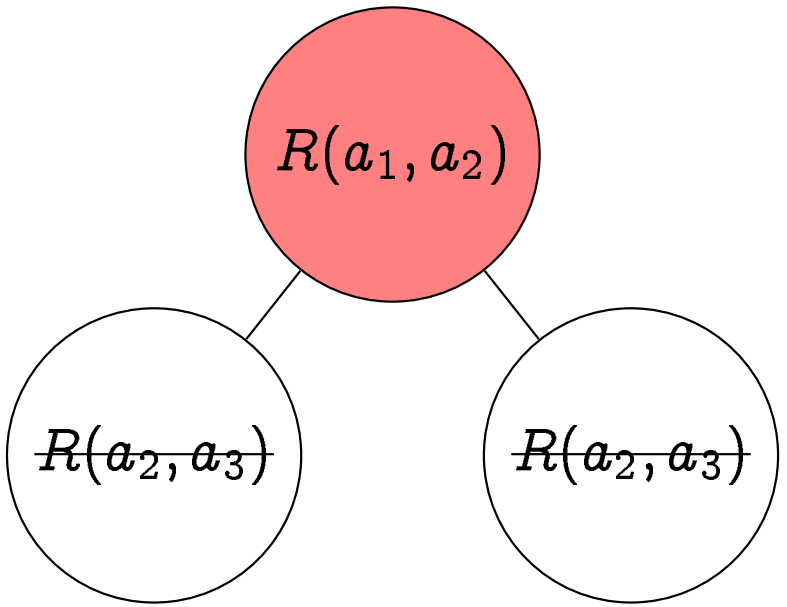

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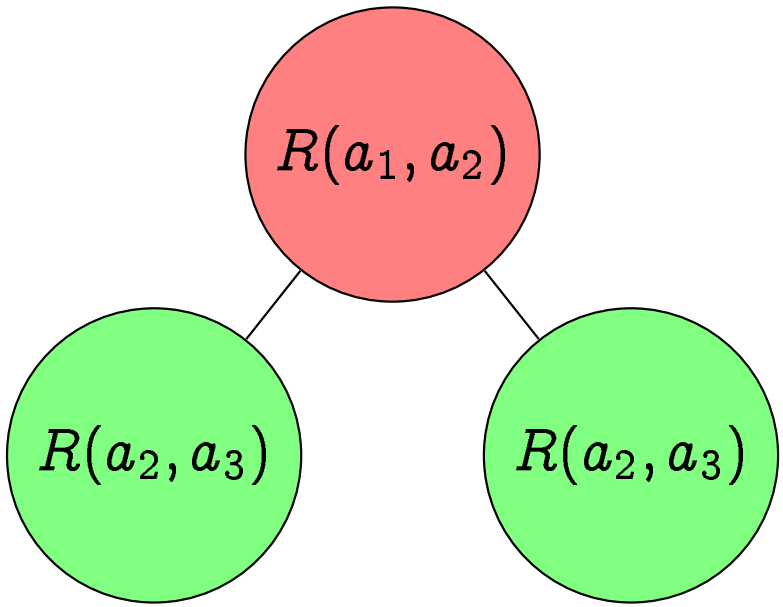

Upper bound on treelike instances

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For any fixed MSO query q and $k \in \mathbb{N}$, for any input instance I of treewidth $\leq k$, we can build in linear time in I a provenance circuit of q on I.

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Corollary (Amarilli et al. (2015))

Probabilistic query evaluation of MSO queries on treelike instances is in linear time in data complexity up to arithmetic operations.

What about combined complexity?

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Theorem (Amarilli et al. (2017a))

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- We can show a 2EXPTIME upper bound on the combined complexity of PQE (Amarilli et al., 2017a) of ICG-Datalog programs of bounded body size on treelike instance (better than the naive non-elementary bound!)

Combined complexity of PQE (2/2)

- We cannot do much better (Amarilli et al., 2017b). We obtain intractability in combined complexity (#P-hardness):
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- Very limited settings of combined tractability of PQE
  - when the instance is a path, and the query is a connected query
  - when the instance is a directed tree, and the query is a directed path

Outline

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  - Otherwise instances of treewidth k in $\mathcal{I}$ could be very large... see (Makowsky and Marino, 2003)

Our lower bound result

Theorem (Amarilli et al. (2016))

There is a first-order query q such that for any unbounded-tw, tw-constructible, arity-2 instance family $\mathcal{I}$, PQE for q on $\mathcal{I}$ is #P-hard under RP reductions.

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- Let G be a planar graph of degree ≤ 3

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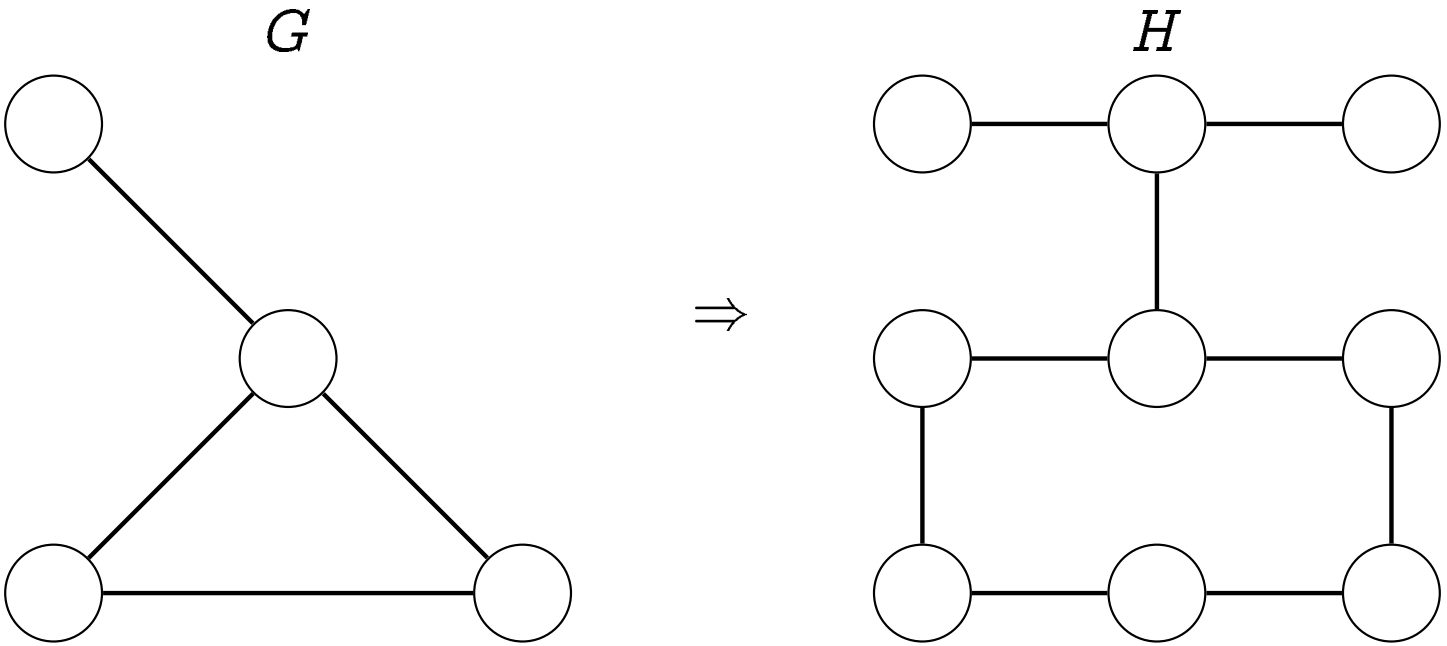

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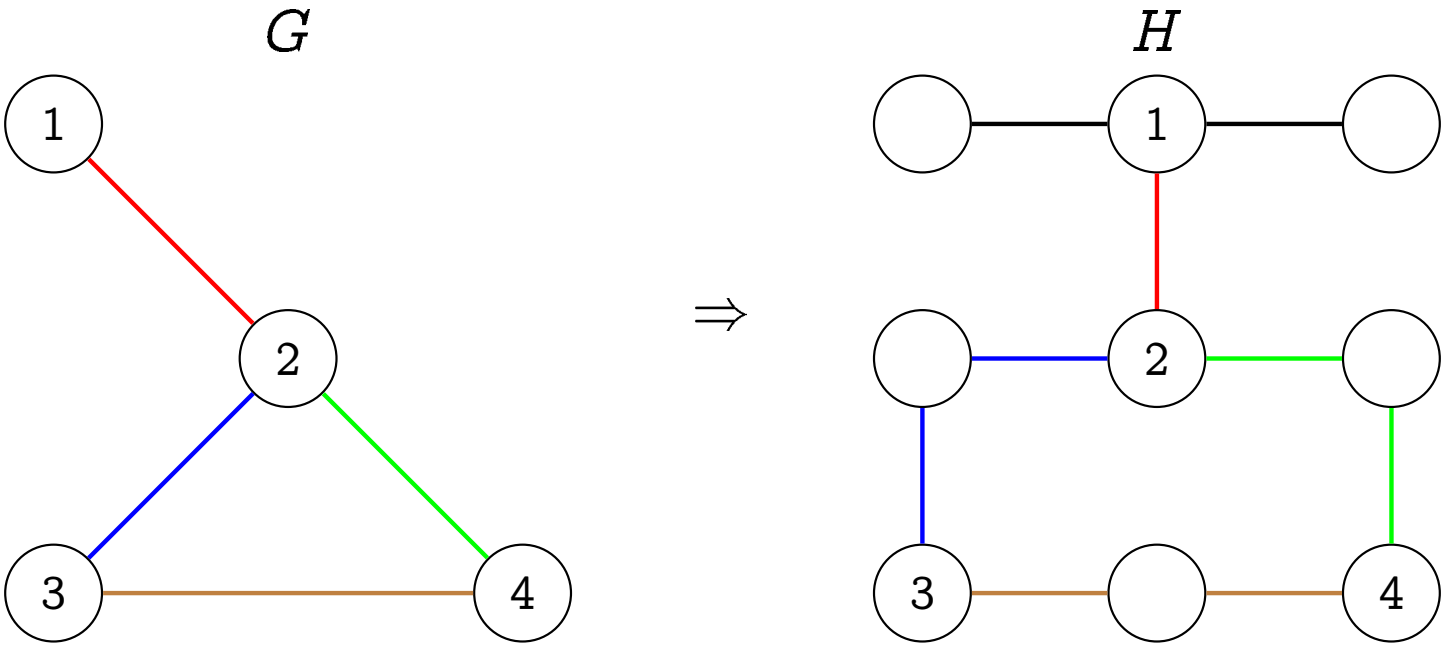

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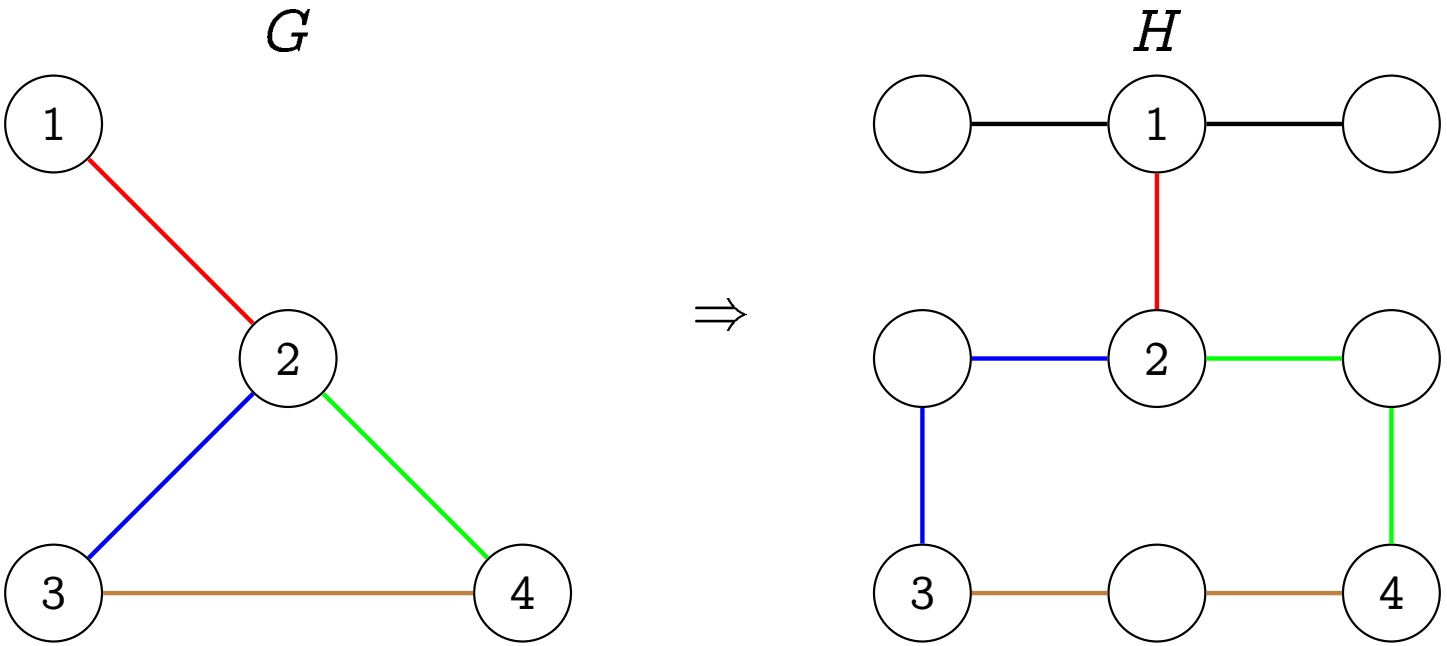

- Map vertices to vertices
- Map edges to vertex-disjoint paths

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More recently:

Theorem (Chekuri and Chuzhoy (2014))

There is a certain constant $c \in \mathbb{N}$ such that
for any planar graph G of degree ≤ 3,
for any graph H of treewidth $\geq |G|^c$,
G is a topological minor of H and
we can embed G in H (w.h.p.) in PTIME in $|H|$.

Intuition for our result: reduction

- Choose a problem from which to reduce:
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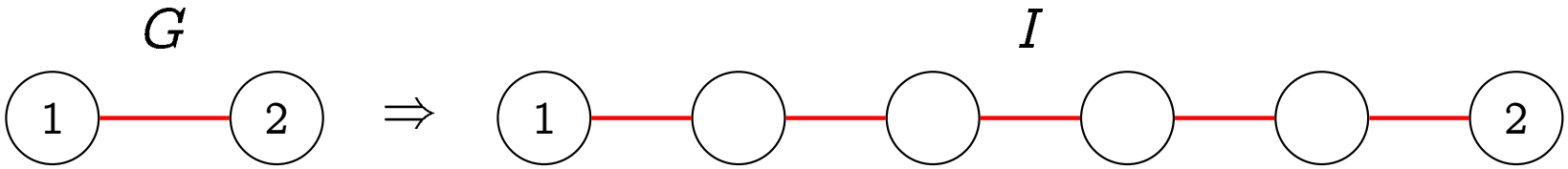
- Construct a probability valuation π of I such that:
  - Unneccessary edges of I are removed
  - PQE for q gives the answer to the hard problem

Technical issue



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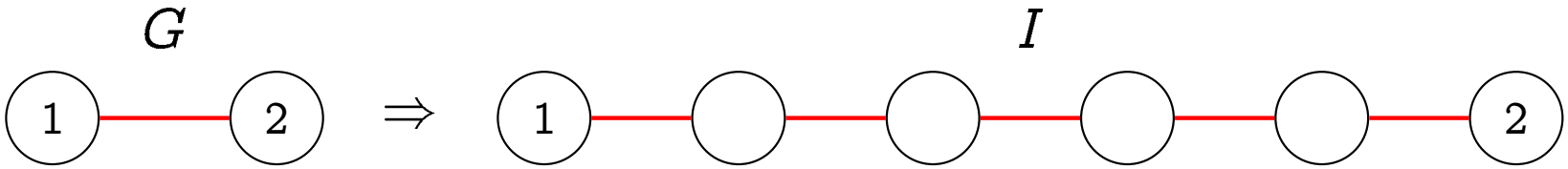

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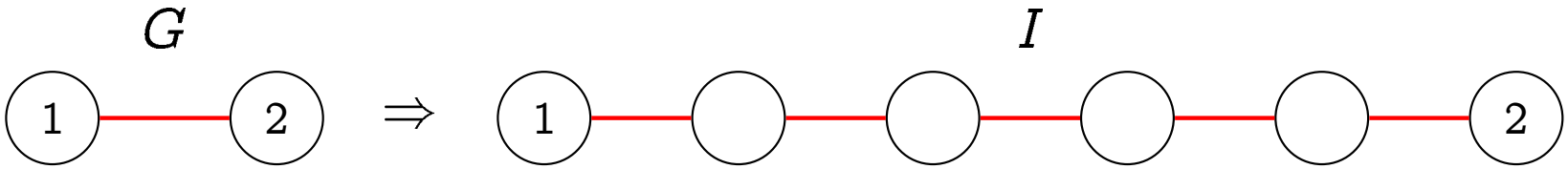

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- For non-probabilistic evaluation, using FO does not work (Frick and Grohe, 2001)
- Lower bounds for non-probabilistic evaluation are for MSO (Ganian et al., 2014)

Can we do better?

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- This UCQ with inequalities is hard in a weaker sense (no polynomial-size OBDD representations of provenance)
- We don't know whether it's #P-hard (because of subdivisions)

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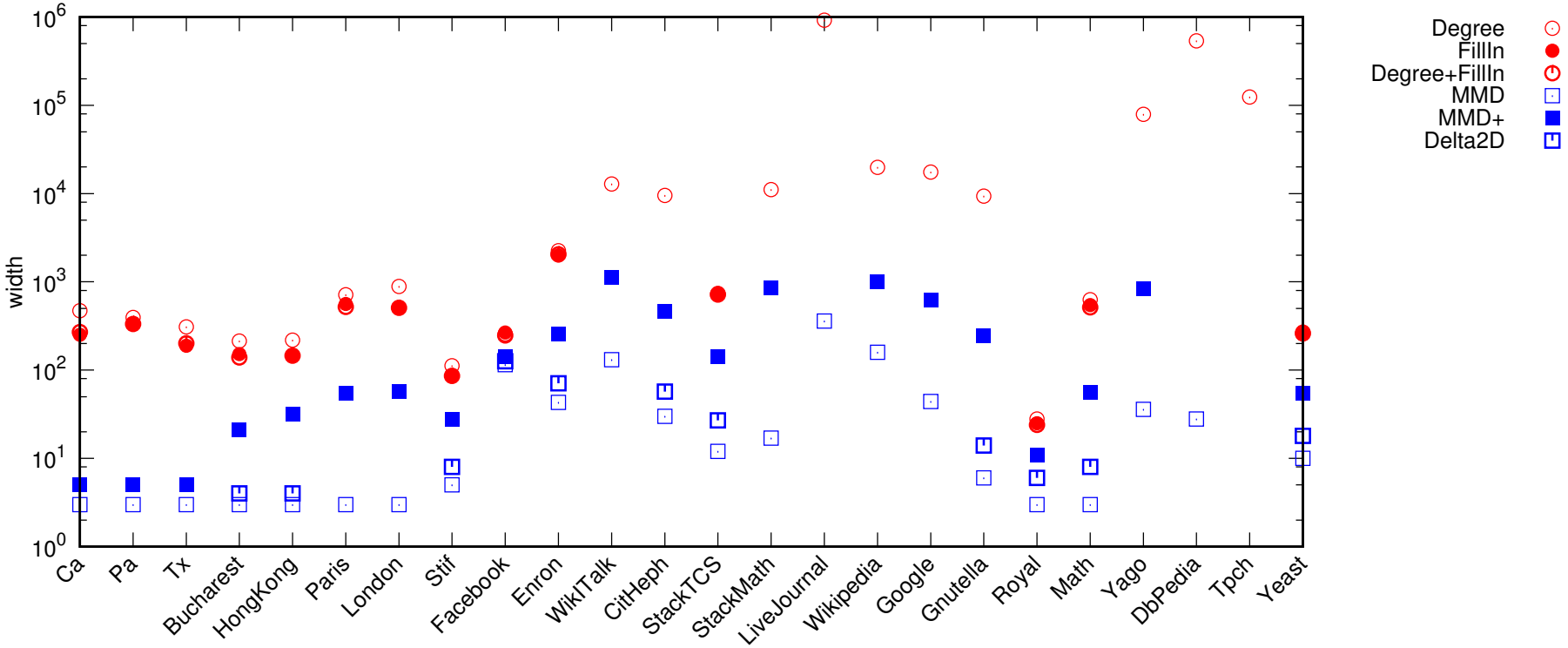
- Are real-world databases low-treewidth?
- If not, can we still do something with them?
- What about approximation algorithms?

Computing the treewidth

- Even computing the treewidth is hard (Arnborg et al., 1987)
- But we can find upper bounds (Bodlaender and Koster, 2010) and lower bounds (Bodlaender and Koster, 2011) on treewidth relatively efficiently
- When we have a bound on the treewidth, we can find a tree decomposition in linear-time (Bodlaender, 1996)...
- but this algorithm is too costly in practice. Better using upper bound algorithms that also provide a tree decomposition

Real-world databases and treewidth (Maniu et al., 2017b)



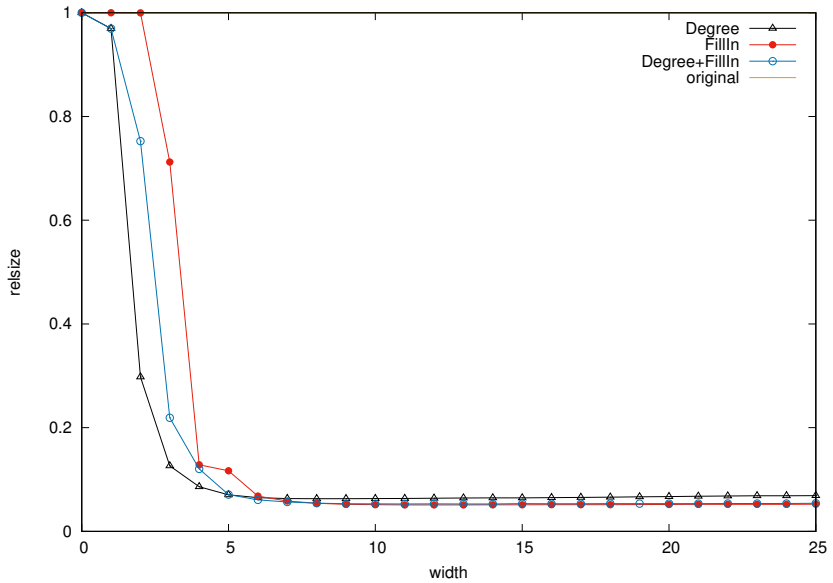
Partial tree decompositions (Maniu et al., 2017a)

If a database has high-treewidth, possible to:

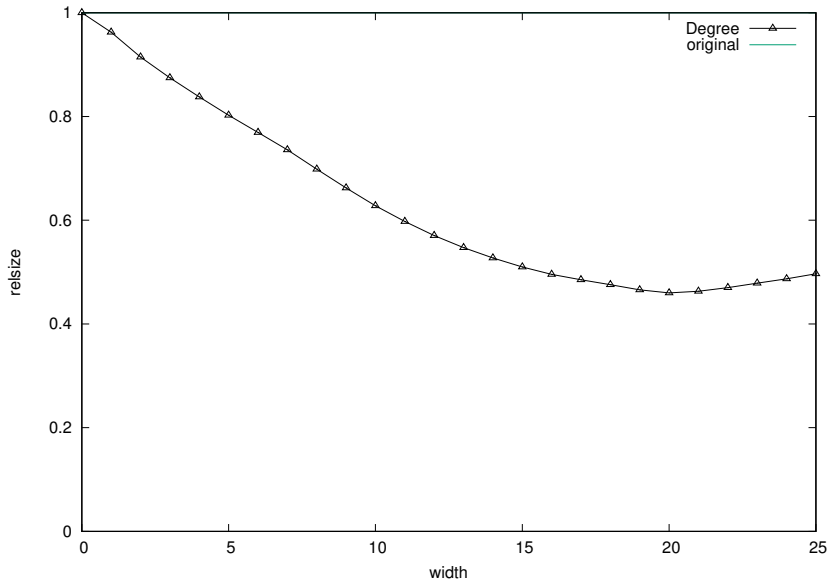
- Isolate a part of low treewidth
- Process this part with efficient PQE techniques
- Process the high-treewidth part (+ whatever is needed to keep track of the low-treewidth part) with other techniques (e.g., approximation algorithms)
- Combine results in a well-founded manner

Partial tree-decomposition of real-world databases

(Maniu et al., 2017b)



OpenStreetMaps Paris
(5m road segments)



Google Web graph fragment
(4m hyperlinks)

Example of query: connectedness (Maniu et al., 2017a)

- Partial tree decomposition with:
  - tendrils of low-treewidth
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Example of query: connectedness (Maniu et al., 2017a)

- Partial tree decomposition with:
  - tendrils of low-treewidth
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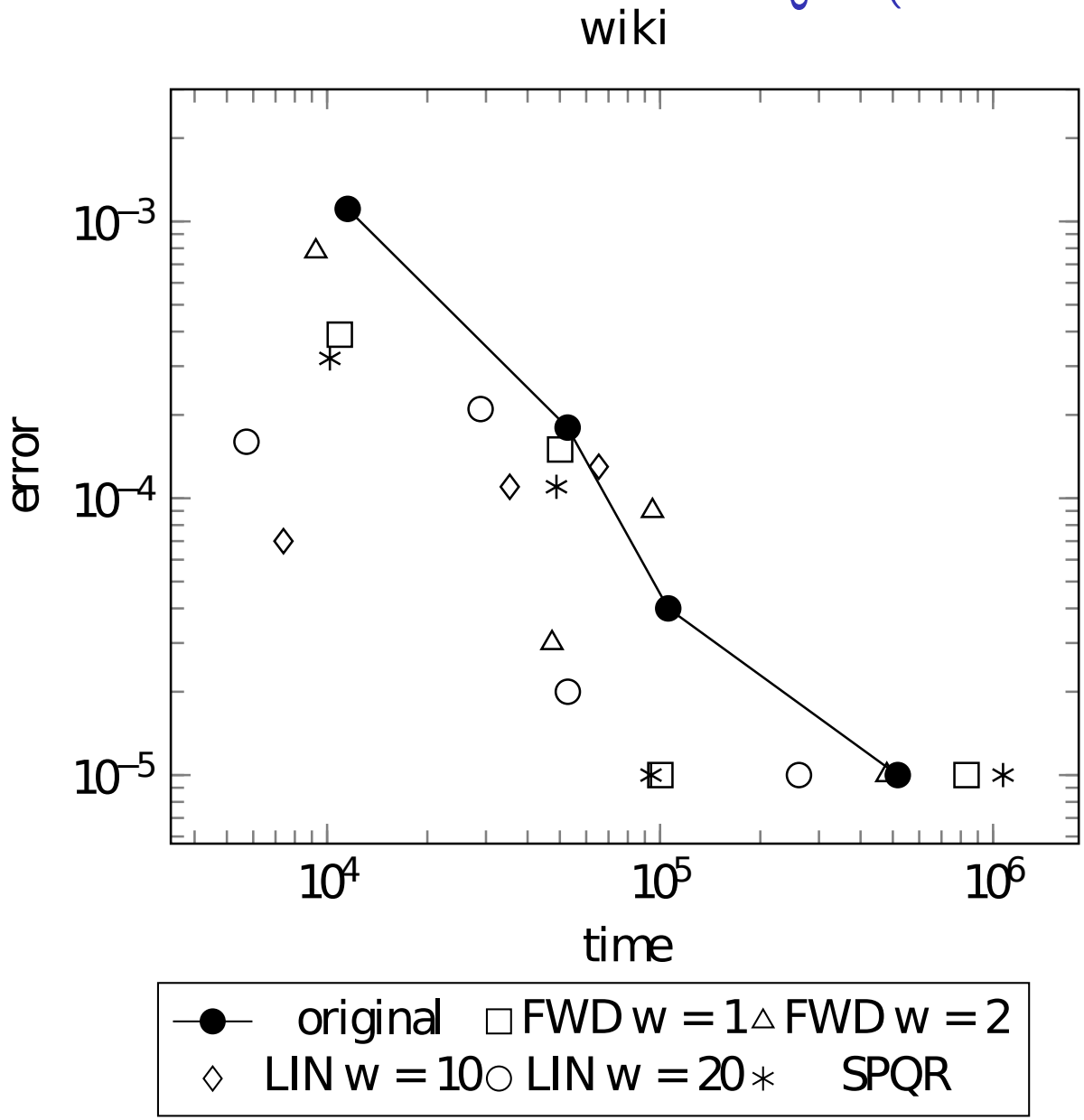
- Algorithm for PQE for the connectedness query:
  - Process the tree decomposition bottom-up, keeping track of the provenance of connectedness between exported nodes
  - Add virtual edges with this provenance as annotation
  - When one reaches the core, use Monte-Carlo sampling to approximate the probability

Performance of connectedness PQE (Maniu et al., 2017a)



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- Always possible to use Monte-Carlo sampling; provide an additive approximation guarantee
- Sometimes (but not always!), possible to use more elaborate approximation algorithms, giving multiplicative approximations (Karp et al., 1989)
- Practical strategy: use query optimization strategies to decide which approximation method to use, possibly on different parts of the data/provenance (Souihli and Senellart, 2013)

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Upper. PQE for MSO on treelike instances
has linear data complexity up to arithmetic costs

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Practical. In practice:

- Real-world databases rarely have low treewidth
- More commonly, a low-treewidth part can be isolated
- Possible to exploit partial tree decompositions for PQE
- One can always resort to approximation algorithms

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- Can we apply all of this to a real-world problem? Routing in public transport networks with a model of uncertainty on schedules?

References I

Amarilli, A., Bourhis, P., Monet, M., and Senellart, P. (2017a). Combined tractability of query evaluation via tree automata and cycluits. In *Proc. ICDT*, pages 6:1–6:19, Venice, Italy.

Amarilli, A., Bourhis, P., and Senellart, P. (2015). Provenance circuits for trees and treelike instances. In *Proc. ICALP*, pages 56–68, Kyoto, Japan.

Amarilli, A., Bourhis, P., and Senellart, P. (2016). Tractable lineages on treelike instances: Limits and extensions. In *Proc. PODS*, pages 355–370, San Francisco, USA.

Amarilli, A., Monet, M., and Senellart, P. (2017b). Conjunctive queries on probabilistic graphs: Combined complexity. In *Proc. PODS*, San Francisco, USA.

References II

Arnborg, S., Corneil, D. G., and Proskuworski, A. (1987). Complexity of finding embeddings in a k-tree. *SIAM Journal on Algebraic and Discrete Methods*, 8(2):277–284.

Bodlaender, H. L. (1996). A linear-time algorithm for finding tree-decompositions of small treewidth. *SIAM Journal on Computing*, 25(6):1305–1317.

Bodlaender, H. L. and Koster, A. M. C. A. (2010). Treewidth computations I. Upper bounds. *Information and Computation*, 208(3):345–353.

Bodlaender, H. L. and Koster, A. M. C. A. (2011). Treewidth computations II. Lower bounds. *Information and Computation*, 209(7):1103–1119.

Chaudhuri, S. and Vardi, M. Y. (1992). On the equivalence of recursive and nonrecursive Datalog programs. In *PODS*.

References III

Chekuri, C. and Chuzhoy, J. (2014). Polynomial bounds for the grid-minor theorem. In *STOC*.

Cohen, S., Kimelfeld, B., and Sagiv, Y. (2009). Running tree automata on probabilistic XML. In *PODS*.

Courcelle, B. (1990). The monadic second-order logic of graphs. I. Recognizable sets of finite graphs. *Inf. Comput.*, 85(1).

Dalvi, N. and Suciu, D. (2012). The dichotomy of probabilistic inference for unions of conjunctive queries. *JACM*, 59(6):30.

Darwiche, A. (2001). On the tractable counting of theory models and its application to truth maintenance and belief revision. *J. Applied Non-Classical Logics*, 11(1-2).

Deutch, D., Milo, T., Roy, S., and Tannen, V. (2014). Circuits for datalog provenance. In *ICDT*.

References IV

Frick, M. and Grohe, M. (2001). Deciding first-order properties of locally tree-decomposable structures. *JACM*, 48(6).

Ganian, R., Hliněný, P., Langer, A., Obdržálek, J., Rossmanith, P., and Sikdar, S. (2014). Lower bounds on the complexity of MSO1 model-checking. *JCSS*, 1(80).

Gottlob, G., Pichler, R., and Wei, F. (2010). Monadic datalog over finite structures of bounded treewidth. *TOCL*, 12(1):3.

Green, T. J., Karvounarakis, G., and Tannen, V. (2007). Provenance semirings. In *PODS*.

Karp, R. M., Luby, M., and Madras, N. (1989). Monte-Carlo approximation algorithms for enumeration problems. *Journal of Algorithms*, 10(3).

References V

Lauritzen, S. L. and Spiegelhalter, D. J. (1988). Local computations with probabilities on graphical structures and their application to expert systems. *J. Royal Statistical Society. Series B.*

Makowsky, J. A. and Marino, J. (2003). Tree-width and the monadic quantifier hierarchy. *Theor. Comput. Sci.*, 303(1).

Maniu, S., Cheng, R., and Senellart, P. (2017a). An indexing framework for queries on probabilistic graphs. *ACM Transactions on Database Systems*, 42(2).

Maniu, S., Senellart, P., and Jog, S. (2017b). Under review.

Riedel, M. D. and Bruck, J. (2012). Cyclic Boolean circuits. *Discrete Applied Mathematics*, 160(13-14).

References VI

Robertson, N. and Seymour, P. D. (1986). Graph minors. V. Excluding a planar graph. *J. Comb. Theory, Ser. B*, 41(1).

Souihli, A. and Senellart, P. (2013). Optimizing approximations of dnf query lineage in probabilistic xml. In *ICDE*.

Thatcher, J. W. and Wright, J. B. (1968). Generalized finite automata theory with an application to a decision problem of second-order logic. *Mathematical systems theory*, 2(1):57–81.

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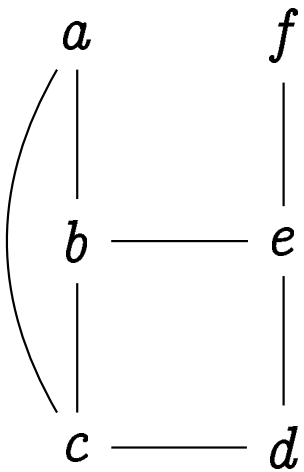

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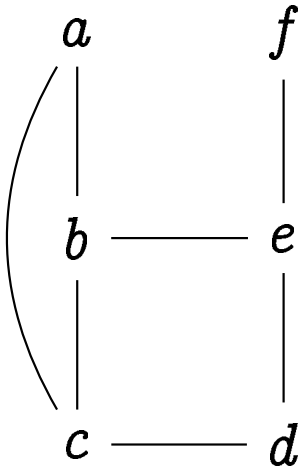

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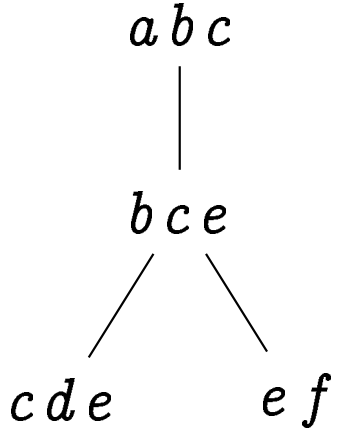

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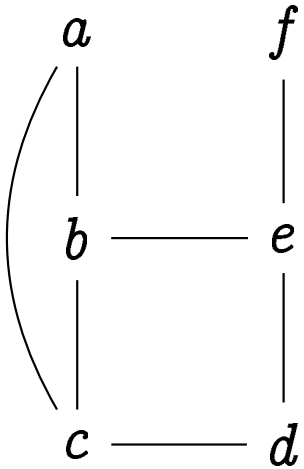

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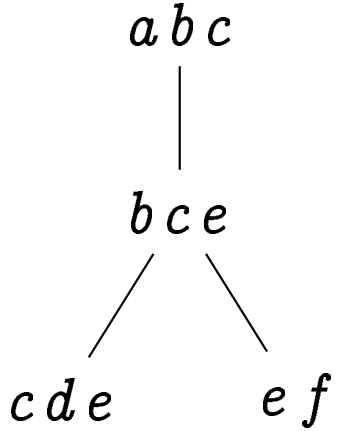

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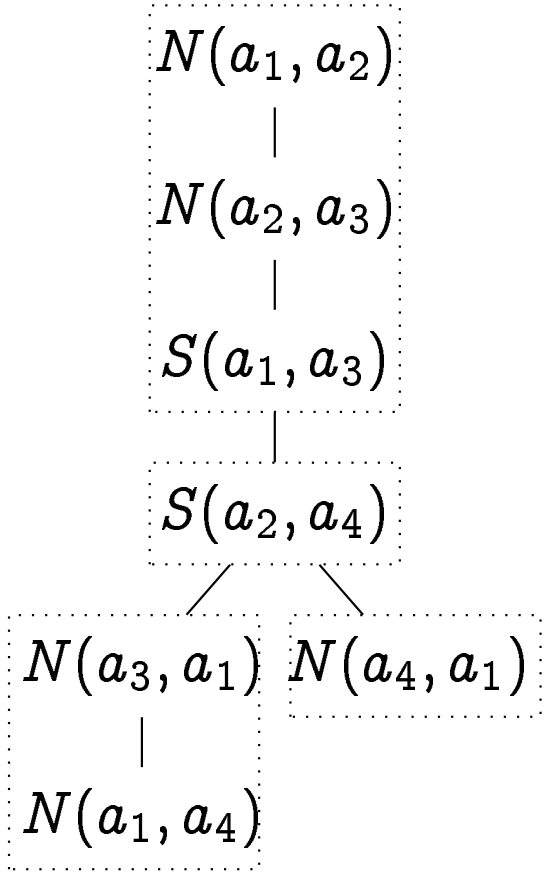

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  - For monotone queries, we can construct positive circuits

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  - counting the number of matches of a query
  - computing the security level of a query result
  - computing the cost of a query result

$\mathbb{N}[X]$-provenance example

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  - Sum over query matches
  - Multiply over matched facts

$\mathbb{N}[X]$-provenance example

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d	e	x_3
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f	f	x_5

$$\exists x\, y\, z\ \ R(x,y) \wedge R(y,z)$$

- PosBool$[X]$-provenance:

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How is $\mathbb{N}[X]$ more expressive than PosBool$[X]$?

- Coefficients: counting multiple matches
- Exponents: using facts multiple times

Capturing $\mathbb{N}[X]$-provenance

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- What fails for MSO and Datalog?
  - Unbounded maximal multiplicity of fact uses

Outline

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More expressive formalism: Block-Independent Disjoint instances:

name	**city**	**iso**	p
pods	san francisco	us	0.8
pods	los angeles	us	0.2
icalp	rome	it	0.1
icalp	florence	it	0.9

pc-tables

More generally, pc-tables to represent arbitrary correlations

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date	teacher	room	
04	John	C42	$\neg x_1$
04	Jane	C42	x_1
11	John	C017	$x_2 \wedge \neg x_1$
11	Jane	C017	$x_2 \wedge x_1$
11	John	C47	$\neg x_2 \wedge \neg x_1$
11	Jane	C47	$\neg x_2 \wedge x_1$

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x_1 John gets sick

- Probability 0.1

x_2 Room C017 is available

- Probability 0.2

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Theorem

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"Tree-like" refers to the underlying instance, adding facts to represent variable occurrences and co-occurrences