

# **Extraction of information in large graphs**

## **Automatic search for synonyms**

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## The dictionary graph

**Computation** (n.) The act or process of computing; calculation; reckoning.

**Computation** (n.) The result of computation; the amount computed.

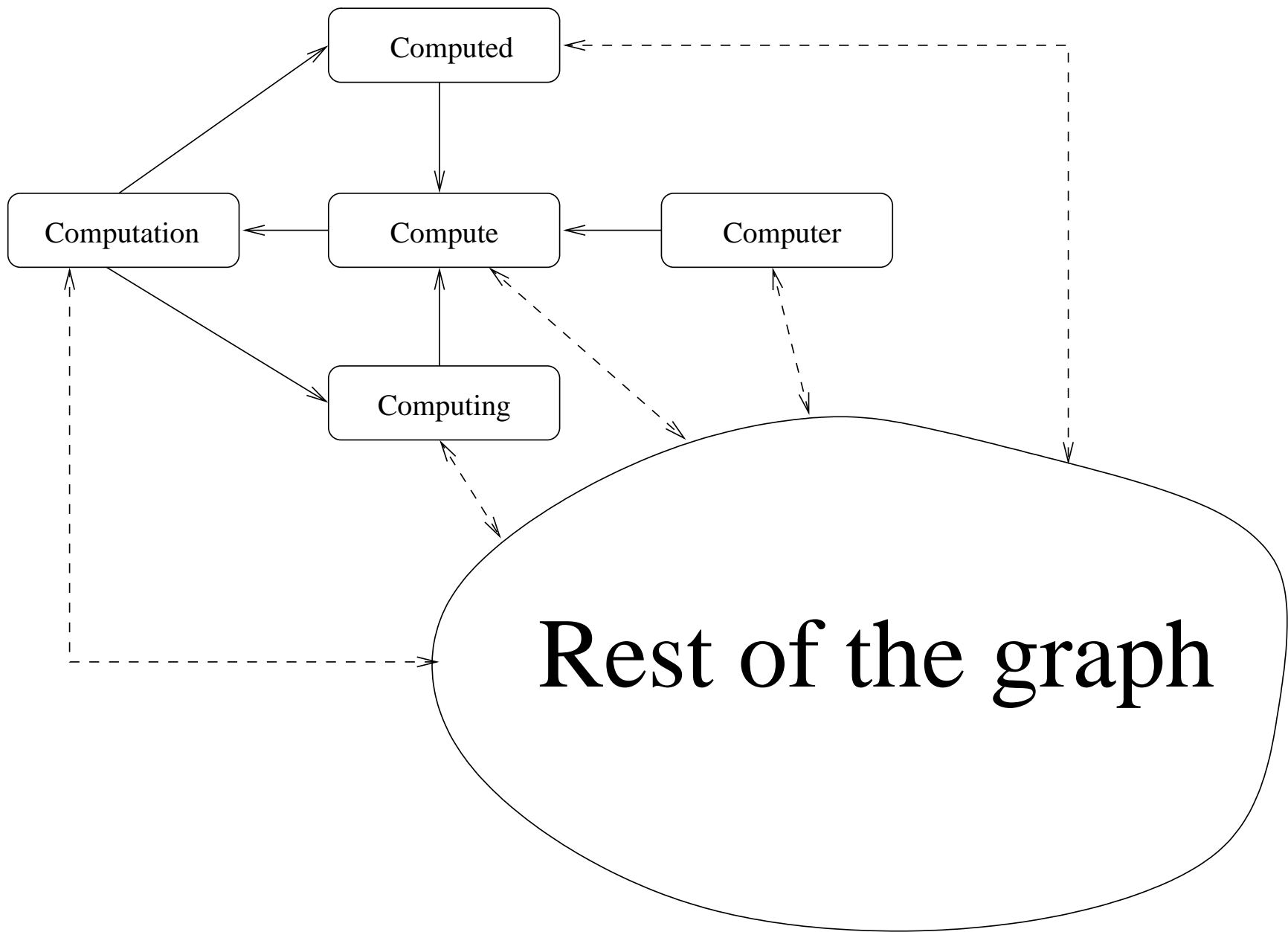
**Computed** (imp. & p. p.) of Compute

**Computing** (p. pr. & vb. n.) of Compute

**Compute** (v.t.) To determine calculation; to reckon; to count.

**Compute** (n.) Computation.

**Computer** (n.) One who computes.



## Extraction of the graph

- Multiwords (e.g. All Saints', Surinam toad)
- Prefixes and suffixes (e.g. un-, -ous)
- Different meanings of a word
- Derived forms (e.g. daisies, sought)
- Accentuated characters (e.g. proven/al, cr/che)
- Misspelled words

112,169 vertices - 1,398,424 arcs.

## Lexical units

13,396 lexical units not defined in the dictionary:

- Numbers (e.g. 14159265, 14th)
- Mathematical and chemical symbols (e.g.  $x^3$ ,  $Fe_3O_4$ )
- Proper nouns (e.g. California, Aaron)
- Misspelled words (e.g. aligator, abudance)
- Undefined words (e.g. snakelike, unwound)
- Abbreviations (e.g. adj, etc)

# Connectivity

185 different connected components:

- 1 111,982-vertex component
- 3 2-vertex components

anguineal → anguineous → snakelike  
indissolvableness → indissolubleness → indissolubility

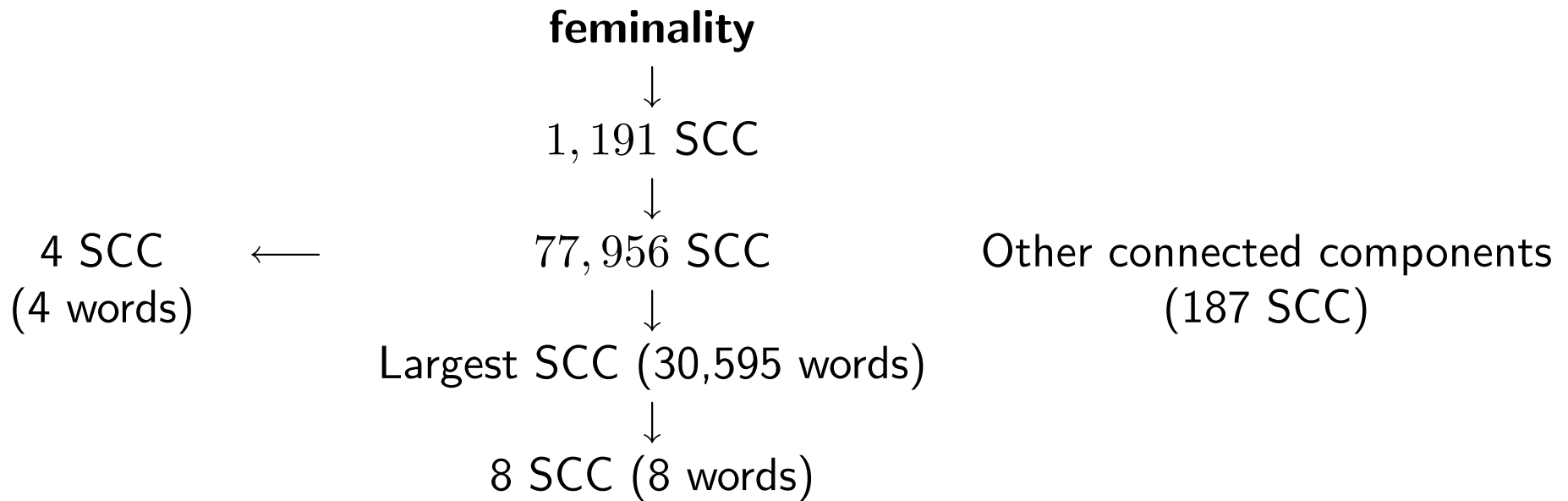
- 181 1-vertex components

## Strong connectivity

79,348 strongly connected components.

Number of vertices	Number of components
30,595	1
10	1
7	3
6	13
5	21
4	50
3	222
2	1,457
1	77,580

Distribution of the size of strongly connected components



Graph resulting of the contraction of each SCC in one single vertex

10-vertex component: **bezpovtsy**, **dukhobors**, **dukhobortsy**, **judaizers**, **molokane**, **molokany**, **popovtsy**, **raskolnik**, **raskolniki**, **skoptsy**.



## A core of the language

**Definition.** A core subgraph of a graph  $G$  is a subgraph of  $G$  with the two following properties:

1. It contains at least one vertex from every directed cycle in  $G$ .
2. Every path in the graph may be prolonged in a path containing a vertex of the subgraph.

If you know the meaning of all the words in a core subgraph of the dictionary graph, you may learn the meaning of all words in the dictionary.

The largest SCC (plus the other connected components and 12 words) is a core subgraph of the dictionary.

## The independence degree

**Definition.** *The independence degree of a graph  $G$  is the minimum number of vertices of a core subgraph of  $G$ .*

**Theorem.** *The computation of the independence degree of a graph is a  $NP$  – complete problem.*

Upper bound :  $30,905 + 187 + 12 = 30,794$ .

Good approximation algorithm?

## A small world

**Definition.** *A graph is a small world graph if it has the following properties:*

- 1. It is undirected, unweighted, sparse and connected.*
- 2. The mean minimal length of a path between any two vertices (which is called the characteristic path length)  $L$  is close to that of a random graph with same  $n$  and  $k$ .*
- 3. The mean over all vertices of the ratio of the number of edges in the neighborhood graph by the number of possible edges in the same subgraph (which is called the clustering coefficient)  $\gamma$  is much greater than that of a random graph of same  $n$  and  $k$ .*

The Web, power distribution graphs, the Kevin Bacon graph are well-known examples of small worlds.

The underlying undirected graph of the largest connected component of the dictionary graph *is* a small world:

1. Obvious.

$$2. L \approx 2,40 \sim 3.61 \approx L_{\text{random}}$$

$$3. \gamma \approx 0.45 \gg 2.19 \cdot 10^{-4} \approx \gamma_{\text{random}}$$

Yet it does not fit the models of small worlds graphs proposed by Duncan J. Watts.

Necessity of a model of directed small worlds?

## Degree distributions

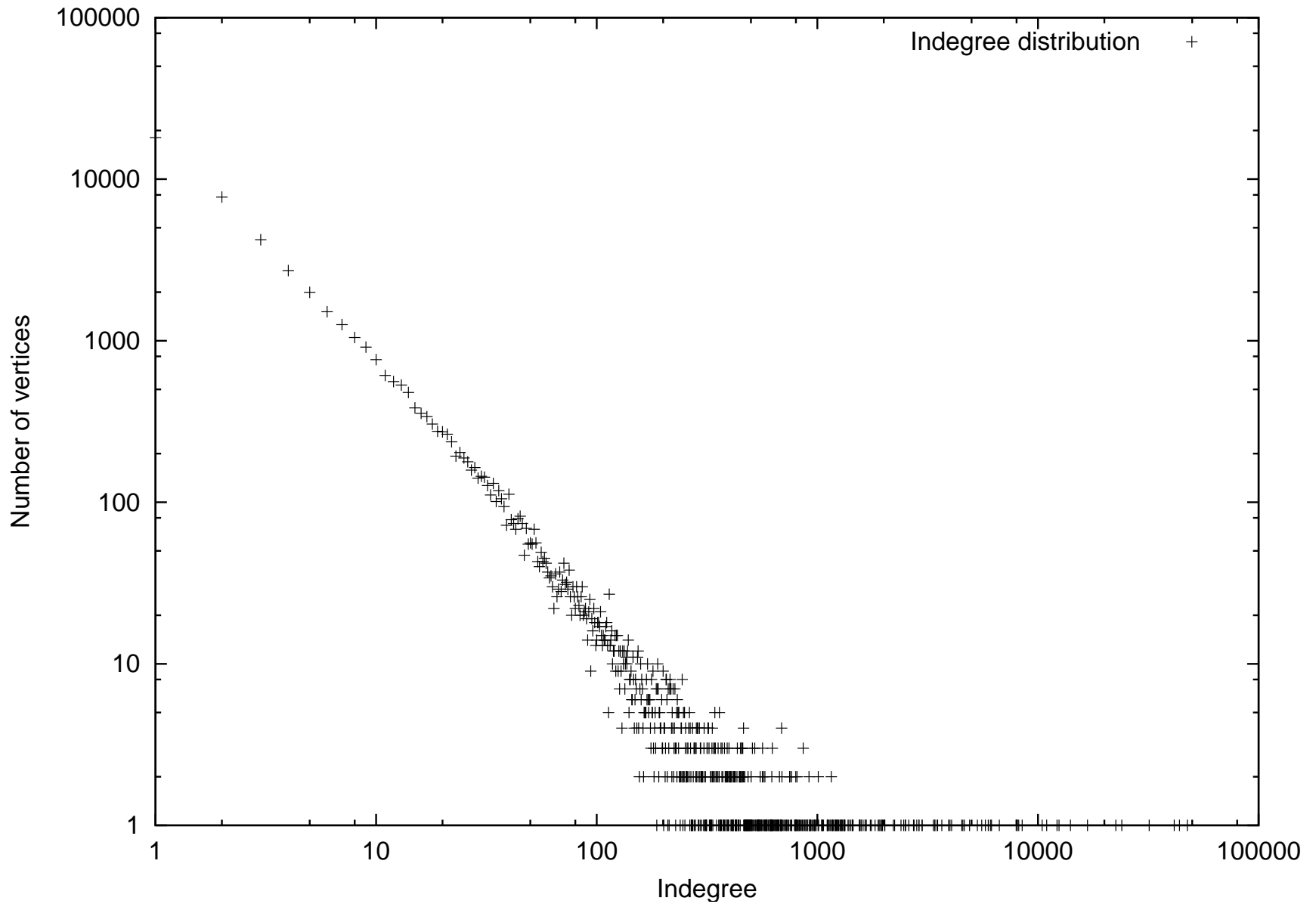
A Zipfian distribution: the probability that a node has indegree or outdegree  $i$  is proportional to  $1/i^\alpha$  for some  $\alpha$ .

Indegree :  $\alpha \approx 1.6$ . Outdegree :  $\alpha \approx 3.1$

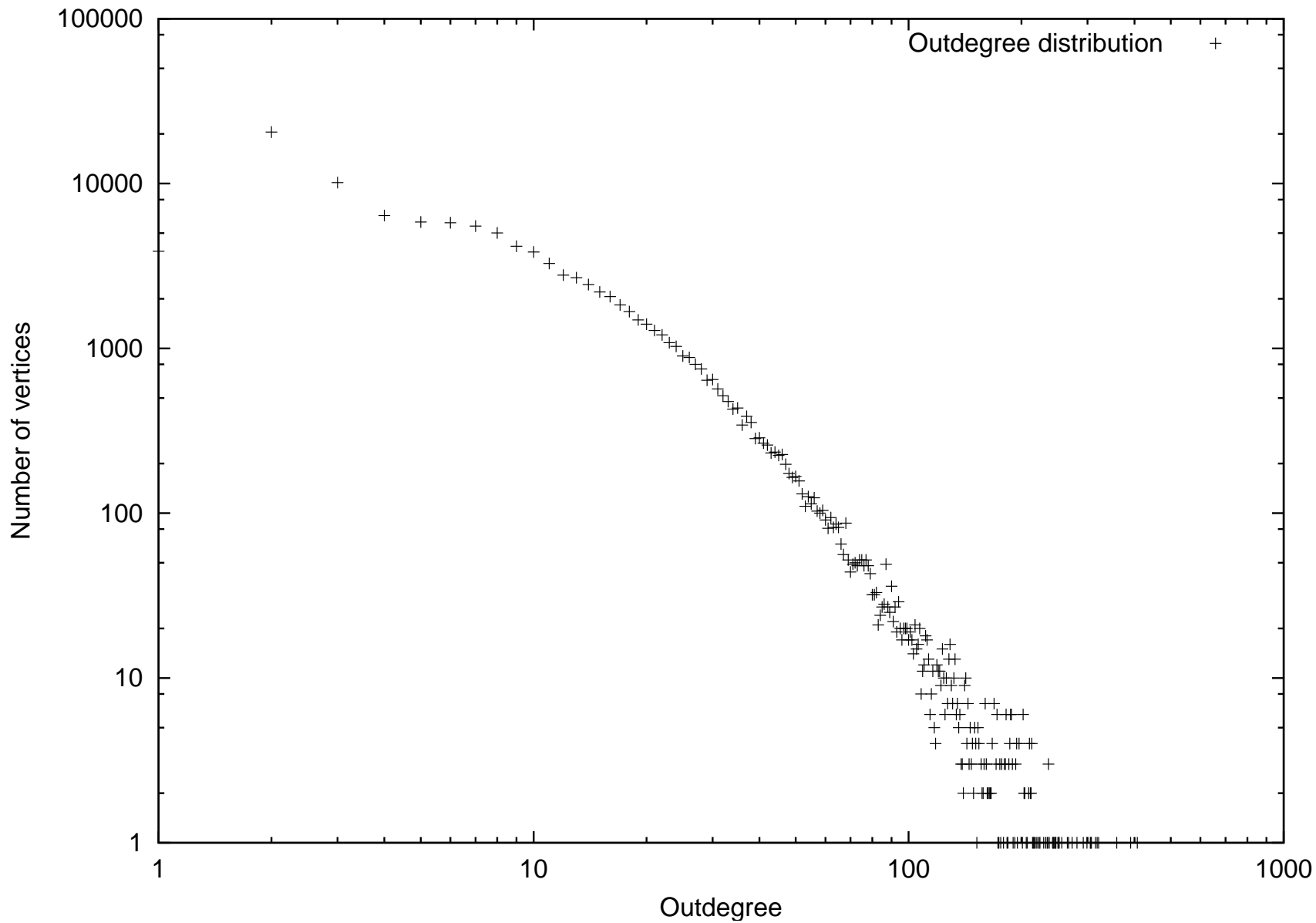
Concerning the outdegree distribution:

1. It is bounded by a rather small amount.
2. The plot is not linear in the range of small outdegrees.

Same kind of distributions as for the Web.



Indegree distribution



Outdegree distribution

## Looking for near-synonyms

**Definition.** *The neighborhood graph of a node  $i$  in a directed graph  $G$  is the subgraph consisting of  $i$ , all parents of  $i$  and all children of  $i$ .*

$i$  is some word we want a synonym of.

$A$  will be the adjacency matrix of the neighborhood graph of  $i$  in the dictionary graph.

$n$  is the order of  $A$ .



## The vectors method

For each  $1 \leq j \leq n, j \neq i$ , compute:

$$\|(A_{i,\cdot} - A_{j,\cdot})\| + \|(A_{\cdot,i} - A_{\cdot,j})^T\|$$

(where  $\| \cdot \|$  is some vector norm,  $A_{i,\cdot}$  and  $A_{\cdot,i}$  are respectively the  $i$ th line and the  $i$ th column of  $A$ ).

For instance, if we choose the Euclidean norm, we compute:

$$\left( \sum_{k=1}^n (A_{i,k} - A_{j,k})^2 \right)^{\frac{1}{2}} + \left( \sum_{k=1}^n (A_{k,i} - A_{k,j})^2 \right)^{\frac{1}{2}}$$

The lower this value is, the best  $j$  is a synonym of  $i$ .

## Kleinberg's algorithm

*Hub*  $\longrightarrow$  *Authority*

A *mutually reinforcing relationship*: good hubs are pages that point to good authorities and good authorities are pages pointed to by good hubs.

The principal eigenvectors of  $A^T A$  and  $AA^T$  give respectively the *authority weights* and *hub weights* of the vertices of the graph.

## An extension of Kleinberg's algorithm

Let  $M(m, m)$  and  $N(n, n)$  be the transition matrices of two oriented graphs.

Let  $C = M \otimes N + M^T \otimes N^T$  where  $\otimes$  is the Kronecker tensorial product.

We assume that the greatest eigenvalue of  $C$  is strictly greater than the absolute value of all other eigenvalues.

Then, the normalized principal eigenvector  $X$  of  $C$  gives the "similarity" between a vertex of  $M$  and a vertex of  $N$ :  $X_{i \times n + j}$  characterizes the similarity between vertex  $i$  of  $M$  and vertex  $j$  of  $N$ .

In particular, if  $M = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ , the result is that of Kleinberg's algorithm.

## Application to the search for synonyms

$$1 \longrightarrow 2 \longrightarrow 3$$

We are looking for vertices “like 2” in the neighborhood graph of  $i$ .

$$\text{Let } C = M \otimes A + M^T \otimes A^T \text{ where } M = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

The principal eigenvector of  $C$  gives the similarity between a node in  $G$  and a node in the graph  $1 \longrightarrow 2 \longrightarrow 3$ .

We just select the subvector corresponding to the vertex 2 in order to have synonymy weights.

## ArcRank

**PageRank** (Google): stationary distribution of weights over vertices corresponding to the principal eigenvector of the adjacency matrix.

**ArcRank:**

$$r_{s,t} = \frac{p_s / |a_s|}{p_t}$$

$|a_s|$  is the outdegree of  $s$ .

$p_t$  is the pagerank of  $t$ .

The best synonyms of  $i$  are the other extremity of the best-ranked arcs arriving to or leaving from  $i$ .

## Disappear

	Vectors	Kleinberg	ArcRank	Wordnet	Microsoft Word
1	<b>vanish</b>	<b>vanish</b>	<b>epidemic</b>	<b>vanish</b>	<b>vanish</b>
2	<b>wear</b>	<b>pass</b>	<b>disappearing</b>	<b>go away</b>	<b>cease to exist</b>
3	<b>die</b>	<b>die</b>	<b>port</b>	<b>end</b>	<b>fade away</b>
4	<b>sail</b>	<b>wear</b>	<b>dissipate</b>	<b>finish</b>	<b>die out</b>
5	<b>faint</b>	<b>faint</b>	<b>cease</b>	<b>terminate</b>	<b>go</b>
6	<b>light</b>	<b>fade</b>	<b>eat</b>	<b>cease</b>	<b>evaporate</b>
7	<b>port</b>	<b>sail</b>	<b>gradually</b>		<b>wane</b>
8	<b>absorb</b>	<b>light</b>	<b>instrumental</b>		<b>expire</b>
9	<b>appear</b>	<b>dissipate</b>	<b>darkness</b>		<b>withdraw</b>
10	<b>cease</b>	<b>cease</b>	<b>efface</b>		<b>pass away</b>

Table 1: Near-synonyms for **disappear**

# Parallelogram

	Vectors	Kleinberg	ArcRank	Wordnet	Microsoft Word
1	<b>square</b>	<b>square</b>	<b>quadrilateral</b>	<b>quadrilateral</b>	<b>diamond</b>
2	<b>parallel</b>	<b>rhomb</b>	<b>gnomon</b>	<b>quadrangle</b>	<b>lozenge</b>
3	<b>rhomb</b>	<b>parallel</b>	<b>right-lined</b>	<b>tetragon</b>	<b>rhomb</b>
4	<b>prism</b>	<b>figure</b>	<b>rectangle</b>		
5	<b>figure</b>	<b>prism</b>	<b>consequently</b>		
6	<b>equal</b>	<b>equal</b>	<b>parallelopiped</b>		
7	<b>quadrilateral</b>	<b>opposite</b>	<b>parallel</b>		
8	<b>opposite</b>	<b>angles</b>	<b>cylinder</b>		
9	<b>altitude</b>	<b>quadrilateral</b>	<b>popular</b>		
10	<b>parallelopiped</b>	<b>rectangle</b>	<b>prism</b>		

Table 2: Near-synonyms for **parallelogram**

# Sugar

	Vectors	Kleinberg	ArcRank	Wordnet	Microsoft Word
1	<b>juice</b>	<b>cane</b>	<b>granulation</b>	<b>sweetening</b>	<b>darling</b>
2	<b>starch</b>	<b>starch</b>	<b>shrub</b>	<b>sweetener</b>	<b>baby</b>
2	<b>cane</b>	<b>sucrose</b>	<b>sucrose</b>	<b>carbohydrate</b>	<b>honey</b>
4	<b>milk</b>	<b>milk</b>	<b>preserve</b>	<b>saccharide</b>	<b>dear</b>
5	<b>molasses</b>	<b>sweet</b>	<b>honeyed</b>	<b>organic compound</b>	<b>love</b>
6	<b>sucrose</b>	<b>dextrose</b>	<b>property</b>	<b>saccarify</b>	<b>dearest</b>
7	<b>wax</b>	<b>molasses</b>	<b>sorghum</b>	<b>sweeten</b>	<b>beloved</b>
8	<b>root</b>	<b>juice</b>	<b>grocer</b>	<b>dulcify</b>	<b>precious</b>
9	<b>crystalline</b>	<b>glucose</b>	<b>acetate</b>	<b>edulcorate</b>	<b>pet</b>
10	<b>confection</b>	<b>lactose</b>	<b>saccharine</b>	<b>dulcorate</b>	<b>babe</b>

Table 3: Near-synonyms for **sugar**



## Science

	Vectors	Kleinberg	ArcRank	Wordnet	Microsoft Word
1	<b>art</b>	<b>art</b>	<b>formulate</b>	<b>knowledge domain</b>	<b>discipline</b>
2	<b>branch</b>	<b>branch</b>	<b>arithmetic</b>	<b>knowledge base</b>	<b>knowledge</b>
3	<b>nature</b>	<b>law</b>	<b>systematize</b>	<b>discipline</b>	<b>skill</b>
4	<b>law</b>	<b>study</b>	<b>scientific</b>	<b>subject</b>	<b>art</b>
5	<b>knowledge</b>	<b>practice</b>	<b>knowledge</b>	<b>subject area</b>	
6	<b>principle</b>	<b>natural</b>	<b>geometry</b>	<b>subject field</b>	
7	<b>life</b>	<b>knowledge</b>	<b>philosophical</b>	<b>field</b>	
8	<b>natural</b>	<b>learning</b>	<b>learning</b>	<b>field of study</b>	
9	<b>electricity</b>	<b>theory</b>	<b>expertness</b>	<b>ability</b>	
10	<b>biology</b>	<b>principle</b>	<b>mathematics</b>	<b>power</b>	

Table 4: Near-synonyms for **science**

## Perspectives

- Extension of the subgraph
- Other dictionaries, other languages
- Other applications of the extension of Kleinberg's algorithm
- A model of small world directed graphs
- Invariants for languages