Une étude expérimentale de la largeur d'arbre de données graphe du monde réel

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BDA 2018

Treewidth: Informal Definition

- Graph-theoretic measure of how close to a tree a graph is
- Computed as the minimum width of a tree decomposition, i.e., a way to build a hierarchy of separators
- Width: maximum size of a separator minus one

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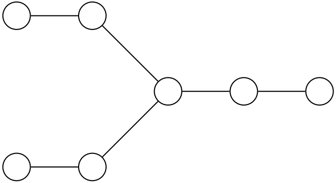

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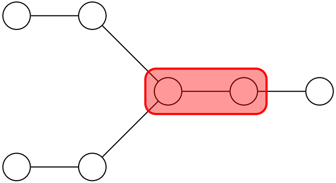

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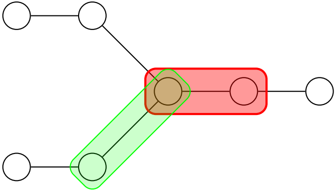

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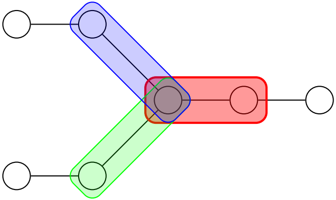

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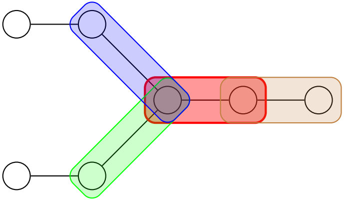

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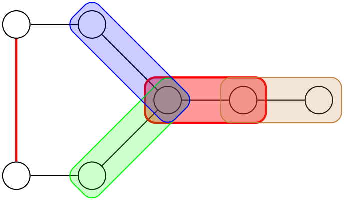

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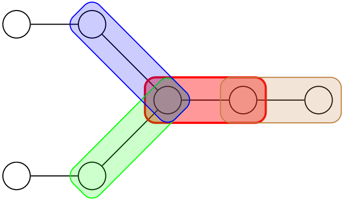

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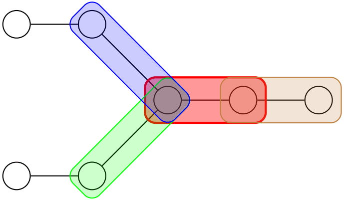

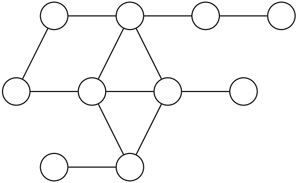

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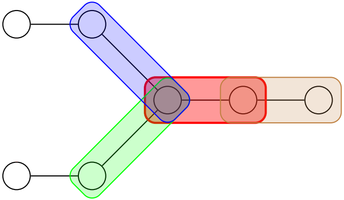

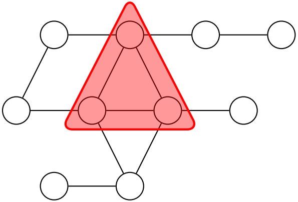

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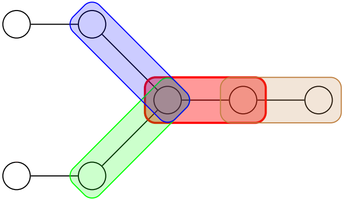

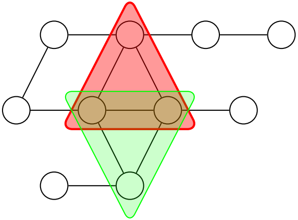

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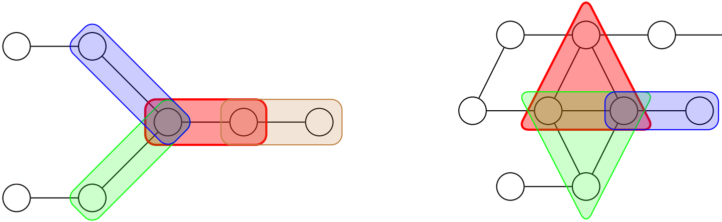

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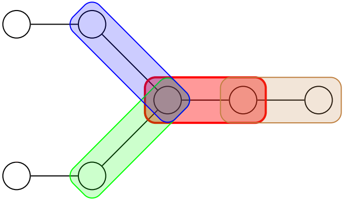

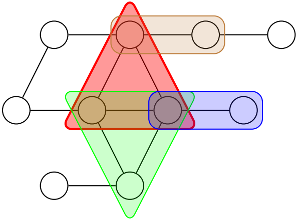

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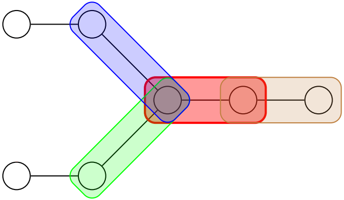

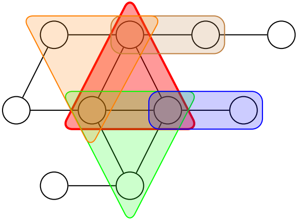

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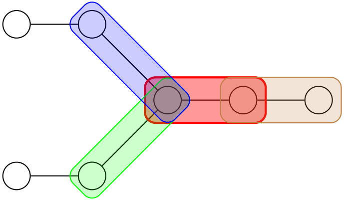

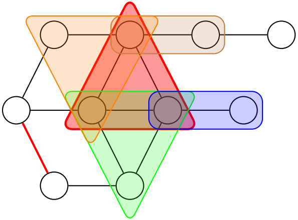

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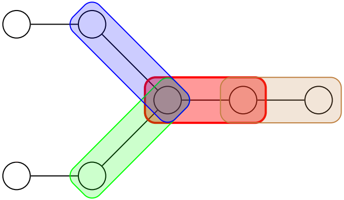

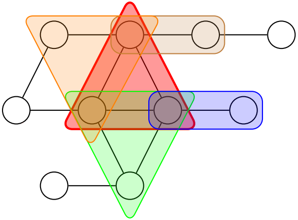

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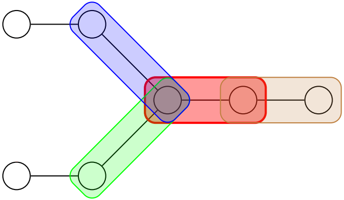

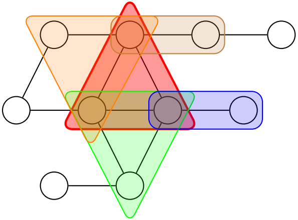

- Trees have treewidth 1
- Cycles have treewidth 2
- k-cliques and $(k-1)$-grids have treewidth $k-1$

Tree decomposition

Definition (Tree decomposition)

A tree decomposition of a graph (V, E) is a pair (T, B) where $T = (I, F)$ is a tree and $B : I \to 2^V$ is a labeling of the nodes of T by subsets of V (called bags), with:

1. $\bigcup_{i \in I} B(i) = V$;
2. $\forall (u, v) \in E$, $\exists i \in I$ s.t. $\{u, v\} \subseteq B(i)$; and
3. $\forall v \in V$, $\{i \in I \mid v \in B(i)\}$ induces a subtree of T.

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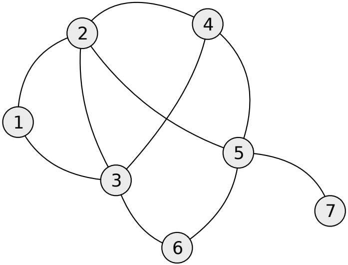

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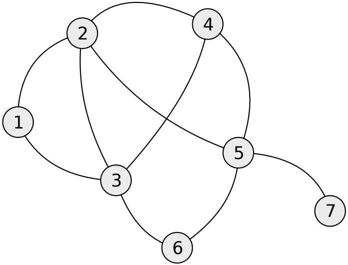

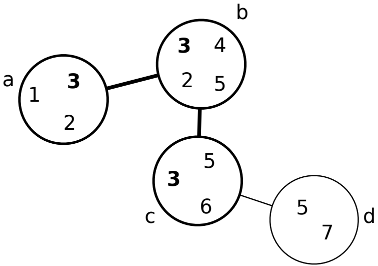

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The width of a tree decomposition is the maximum size of a bag in it, minus one. The treewidth of a graph is the minimum width of a tree decomposition of this graph.

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In databases:

- Readily usable notion for graph databases (treewidth of the underlying graph)
- Treewidth of a relational database: that of its Gaifman graph (the graph where data values are nodes, and two data values are connected if they co-occur in the same tuple)

Tree Decompositions of Relational Data

Instance:

N	
a	b
b	c
c	d
d	e
e	f

S	
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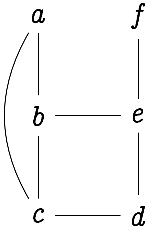
Gaifman graph:



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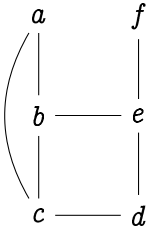

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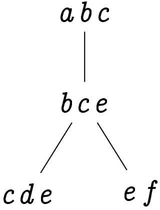

Outline

Treewidth

Motivation

Treewidth Computation

Treewidth of Real-World Data

Conclusion

Complex Query Evaluation is Hard!

- query evaluation of Boolean monadic second-order (MSO) queries is hard for every level of the polynomial hierarchy (Ajtai et al., 2000);
- unless P = NP, there is no polynomial-time counting or enumeration algorithm for first-order (FO) queries with free second-order variables (Saluja et al., 1995; Durand and Strozecki, 2011);
- computing the probability of conjunctive queries (CQs) over tuple-independent databases is #P-hard (Dalvi and Suciu, 2007);
- unless P = NP, there is no polynomial-time algorithm to construct a deterministic decomposable negation normal form (d-DNNF) representation of the Boolean provenance of some CQ (Dalvi and Suciu, 2007; Jha and Suciu, 2013).

Low Treewidth Makes Things Easy!

Assume we know that the databases we work with have treewidth less than some fixed constant k. Then:

- query evaluation of MSO queries is linear-time (Courcelle, 1990; Flum et al., 2002);
- counting (Arnborg et al., 1987) and enumeration (Bagan, 2006; Amarilli et al., 2017) of MSO queries is linear-time;
- computing the probability of MSO queries over a bounded-treewidth tuple-independent database is linear-time assuming constant-time rational arithmetic (Amarilli et al., 2015);
- a d-DNNF representation of the provenance of any MSO query can be computed in linear time (Amarilli et al., 2016).

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(These algorithms are hiding a non-elementary dependency in k, so only feasible for very low values of k.)

Low Treewidth: Only Hope?

- In some cases, there are other ways to have low complexity:

  Query evaluation of MSO queries is linear-time over bounded-cliquewidth databases. (Courcelle et al., 2000)

- But in others, there are none!

  There exists an FO-query Q such that for any unbounded-treewidth family of databases $\mathcal{D}$, probabilistic query evaluation of Q over $\mathcal{D}$ is #P-hard under RP reductions (assuming arity is 2, and some technical condition). (Amarilli et al., 2016)

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- Are real-world databases low-treewidth?
- If not, can we still do something with them?

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Computing the Treewidth

- Even computing the treewidth is hard (Arnborg et al., 1987)
- But we can find upper bounds (Bodlaender and Koster, 2010) and lower bounds (Bodlaender and Koster, 2011) on treewidth relatively efficiently
- When we have a bound on the treewidth, we can find a tree decomposition in linear-time (Bodlaender, 1996)...
- but this algorithm is too costly in practice. Better use upper bound algorithms that also provide a tree decomposition

Upper Bound Algorithms (Bodlaender and Koster, 2010)

- General strategy:
  - Choose an ordering strategy between nodes (e.g., start with nodes with low degree)
  - Eliminate nodes in this order
  - As nodes are eliminated, put remaining neighbors in a bag and add edges between them so that they form a clique
- The resulting procedure constructs a tree decomposition of the graph
- Algorithms differ by their choice of ordering strategy:
  - minimum degree first
  - minimum fill-in first (# edges to add)
  - combination of both

Lower Bound Algorithms (Bodlaender and Koster, 2011)

- Use a proxy that is proved to be always lower than the treewidth:
  - Second lowest degree
  - Second lowest degree in a subgraph of the graph
  - Second lowest degree in a minor of the graph
- Algorithms differ in the way they explore subgraphs or minors (usually greedily):
  - by removing nodes of smallest degree
  - by removing nodes of smallest degree except for a fixed node, and trying all such fixed nodes
  - by contracting edges incident to nodes of smallest degree

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Experimental Setup

- 25 datasets from 8 different domains
- All tests ran on a machine with 32GB RAM, Intel Xeon 1.70GHz CPU
- Up to two weeks of computation time before termination

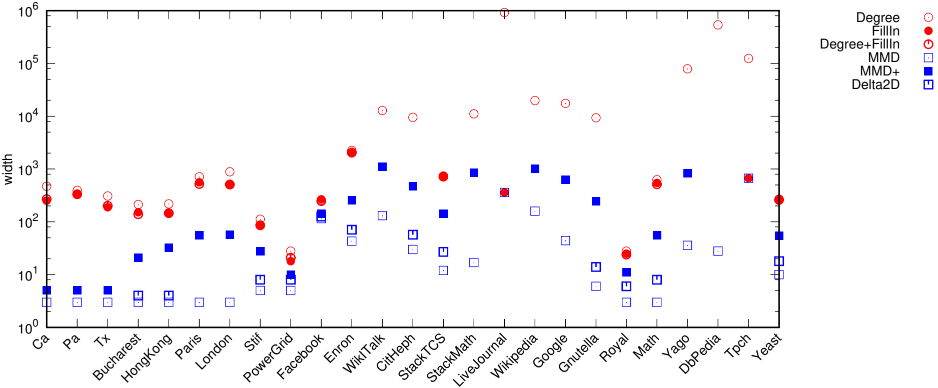
Datasets (1/2)

type	name	nodes	edges
infrastructure	CA	1 965 206	2 766 607
	PA	1 088 092	1 541 898
	TX	1 379 917	1 921 660
	BUCHAREST	189 732	223 143
	HONGKONG	321 210	409 038
	PARIS	4 325 486	5 395 531
	LONDON	2 099 114	2 588 544
	STIF	17 720	31 799
	USPOWERGRID	4 941	6 594
social	FACEBOOK	4 039	88 234
	ENRON	36 692	183 831
	WIKITALK	2 394 385	4 659 565
	CITHEPH	34 546	420 877

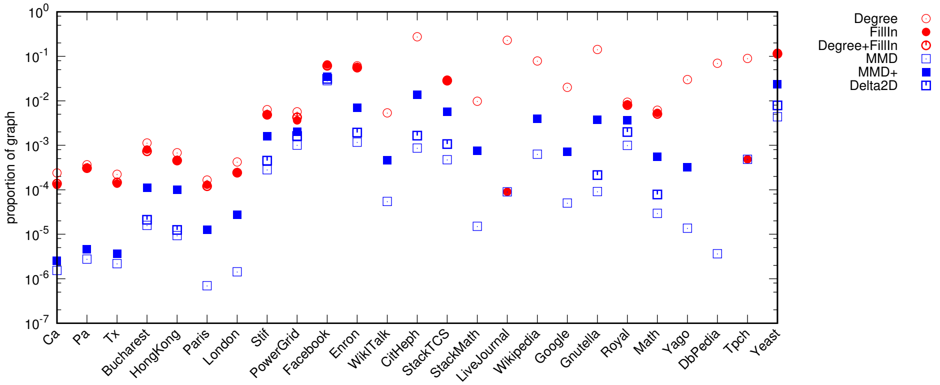
Datasets (2/2)

social	Stack-TCS	25 232	69 026
	Stack-Math	1 132 468	2 853 815
	LiveJournal	3 997 962	34 681 189
web	Wikipedia	252 335	2 427 434
	Google	875 713	4 322 051
communication	Gnutella	65 586	147 892
hierarchy	Royal	3 007	4 862
	Math	101 898	105 131
ontology	Yago	2 635 315	5 216 293
	DbPedia	7 697 211	30 622 392
database	Tpch	1 381 291	79 352 127
biology	Yeast	2 284	6 646

Lower and Upper Bounds (Absolute)



Lower and Upper Bounds (Relative)

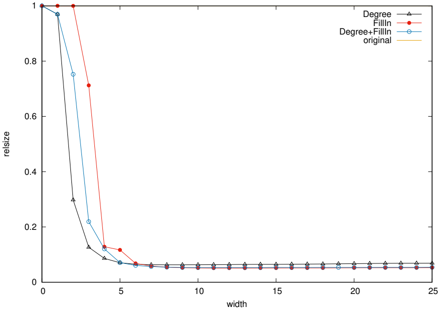


Partial Tree Decompositions

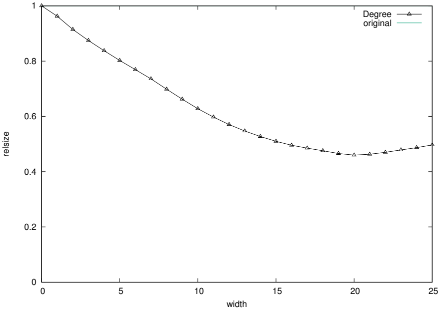
If a database has high-treewidth, possible to:

- Isolate a part of low treewidth
- Process this part with efficient techniques
- Process the high-treewidth part (+ whatever is needed to keep track of the low-treewidth part) with other techniques (e.g., approximation algorithms)
- Combine results in a well-founded manner

Partial Tree Decomposition Results



OpenStreetMaps Paris
(5m road segments)



Google Web graph fragment
(4m hyperlinks)

Example Application: Probability of Connectedness

(Maniu et al., 2017)

- Partial tree decomposition with:
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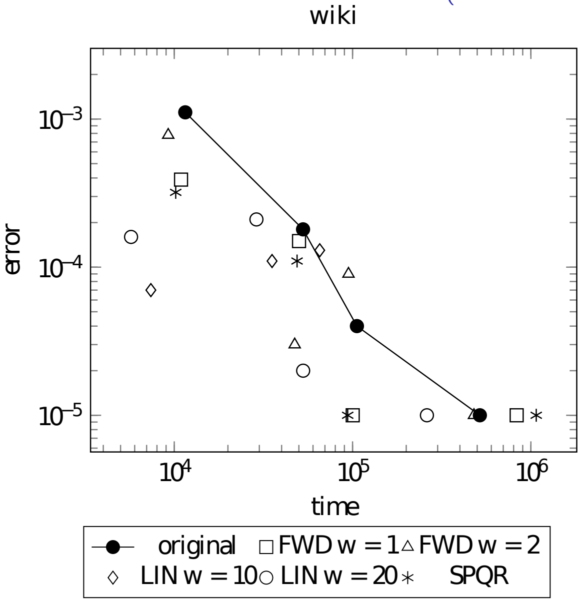
- Partial tree decomposition with:
  - tendrils of low-treewidth
  - a root node of high-treewidth
- Algorithm for probabilistic query evaluation for the connectedness query:
  - Process the tree decomposition bottom-up, keeping track of the provenance of connectedness between exported nodes
  - Add virtual edges with this provenance as annotation
  - When one reaches the core, use Monte-Carlo sampling to approximate the probability

Performance for Connectedness (Maniu et al., 2017)



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- Big gap between upper and lower bounds on treewidth
- Also in this work:
  - More experimental results
  - Comparative running time of different upper and lower bound algorithms

Summary

- Treewidth is never low (<10) ☹
- Infrastructure network have treewidth lower than other kind of networks: $O(\sqrt[3]{n})$?
- Partial tree decompositions can be very effective
- Big gap between upper and lower bounds on treewidth
- Also in this work:
    - More experimental results
    - Comparative running time of different upper and lower bound algorithms
    - Partial tree decompositions of synthetic graph models

Open Questions and Future Work

- Can we formally prove results on complexity of complex query answering based on parameters of partial tree decompositions?

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- Can we extend the connectedness algorithm on partial tree decompositions to more interesting query languages (regular path queries)? To more general notions of provenance?
- Can we apply all of this to a real-world problem? Routing in public transport networks with a model of uncertainty on schedules?

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