Conjunctive Queries on Probabilistic Graphs: Combined Complexity

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Tuple-independent databases (TID)

- Probabilistic databases: model uncertainty about data
- Simplest model: tuple-independent databases (TID)
  - A relational database I
  - A probability valuation π mapping each fact of I to $[0, 1]$
- Semantics of a TID (I, π): a probability distribution on $I' \subseteq I$:
  - Each fact $F \in I$ is either present or absent with probability $\pi(F)$
  - Assume independence across facts

Example: TID

S		
a	b	.5
a	c	.2

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a	b
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$.5 \times .2$

S	
a	b
a	c

$.5 \times (1 - .2)$

S	
a	b

Example: TID

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$(1 - .5) \times .2$

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a	c

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a	b

$(1 - .5) \times .2$

S	
a	c

$(1 - .5) \times (1 - .2)$

S

Probabilistic query evaluation (PQE)

Let us fix:

- Relational signature σ
- Class $\mathcal{I}$ of relational instances on σ (e.g., acyclic, treelike)
- Class $\mathcal{Q}$ of Boolean queries (e.g., paths, trees)

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Probabilistic query evaluation (PQE) problem for $\mathcal{Q}$ and $\mathcal{I}$:

- Given a query $q \in \mathcal{Q}$
- Given an instance $I \in \mathcal{I}$ and a probability valuation π
- Compute the probability that (I, π) satisfies q

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$\rightarrow$ $\Pr((I, \pi) \models q) = \sum_{J \subseteq I,\ J \models q} \Pr(J)$

Complexity of probabilistic query evaluation (PQE)

Question: what is the (data, combined) complexity of PQE depending on the class $\mathcal{Q}$ of queries and class $\mathcal{I}$ of instances?

Data complexity results

- Existing data dichotomy result on queries [Dalvi & Suciu, 2012]
  - $\mathcal{Q}$ = UCQs
  - $\mathcal{I}$ is all instances
  - There is a class $\mathcal{S} \subseteq \mathcal{Q}$ of safe queries

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→ PQE is **#P-hard** for any $q \in \mathcal{Q} \backslash \mathcal{S}$

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What about combined complexity?

Restrict to CQs on graph signatures

$\exists x\, y\, z\, t\; R(x,y) \wedge S(y,z) \wedge S(t,z)$

R		
a	b	.1
b	c	.1
c	d	.05
d	a	1.
d	b	.8

S		
b	d	.7

Restrict to CQs on graph signatures

$\exists x\, y\, z\, t\; R(x, y) \wedge S(y, z) \wedge S(t, z) \quad \rightarrow \quad x \xrightarrow{R} y \xrightarrow{S} z \xleftarrow{S} t$

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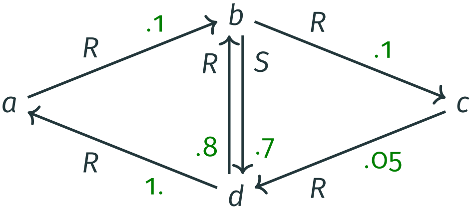

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Restrict instances to trees

$\mathcal{Q}$ = one-way paths (1WP), $\mathcal{I}$ = polytrees (PT)

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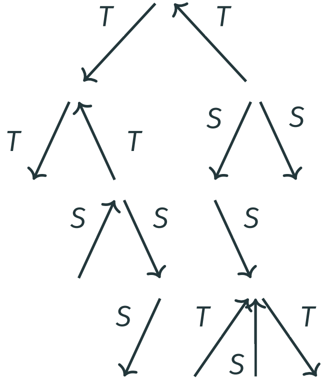
I:



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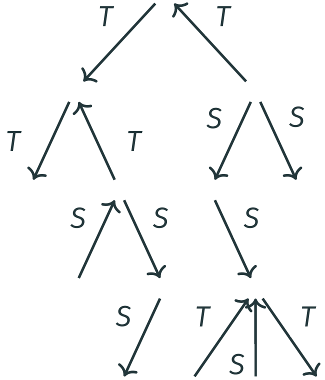

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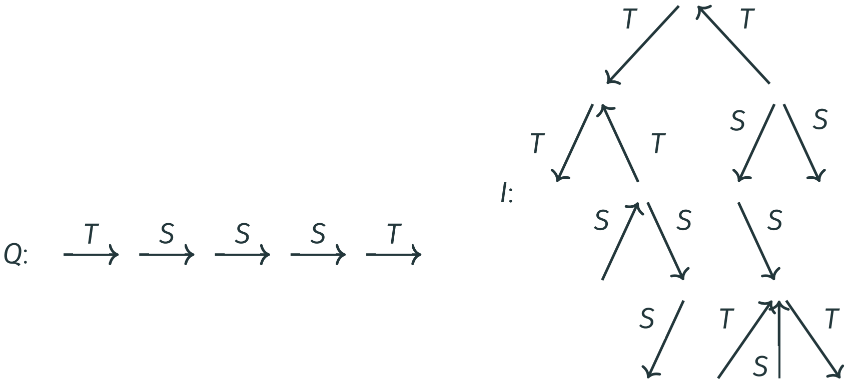
Proposition

PQE of 1WP *on* PT *is* ***#P-hard***

$\mathcal{Q}$ = *one-way paths*, $\mathcal{I}$ = *polytrees*, without labels

- What if we do not have labels?



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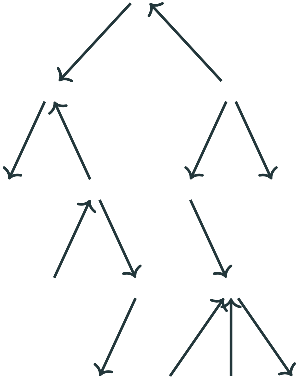

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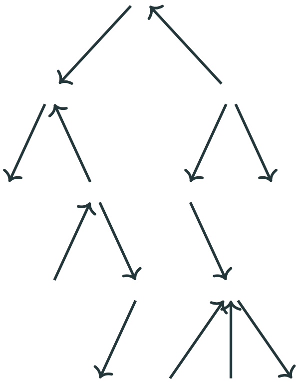

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- PTIME: Bottom-up, e.g., tree automaton

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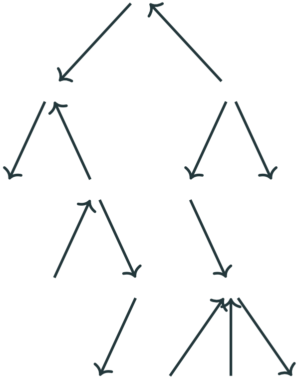

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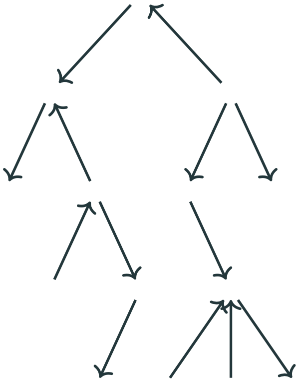
- What if we do not have labels?
- Probability that the instance graph has a path of length $|Q|$
- PTIME: Bottom-up, e.g., tree automaton
- Labels have an impact!

I:



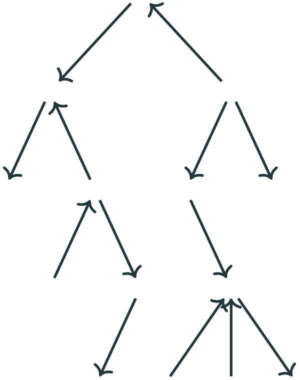
Q:

+ prob. for each edge

$\mathcal{Q}$ = *two-way paths*, $\mathcal{I}$ = *polytrees*, without labels

- $\mathcal{Q}$ = one-way paths (1WP), $\mathcal{I}$ = polytrees (PT)

I:



Q:

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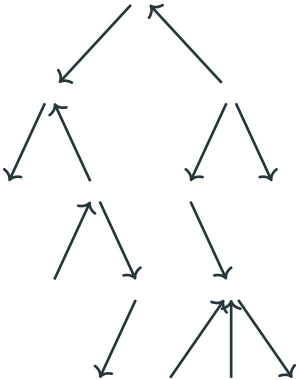
- $\mathcal{Q}$ = **two**-way paths (2WP), $\mathcal{I}$ = polytrees (PT)

I:



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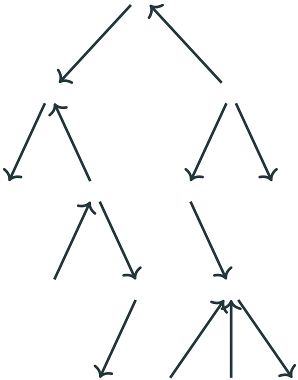

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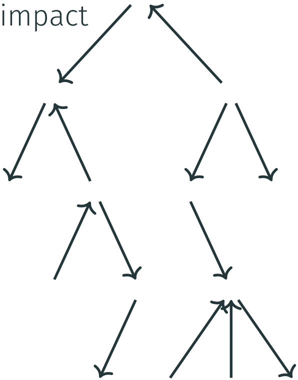
- $\mathcal{Q}$ = **two**-way paths (2WP), $\mathcal{I}$ = polytrees (PT)
- **#P-hard**
- Global orientation of the query has an impact

Q: $\leftarrow \; \rightarrow \; \rightarrow \; \leftarrow \; \leftarrow$

I:



+ prob. for each edge

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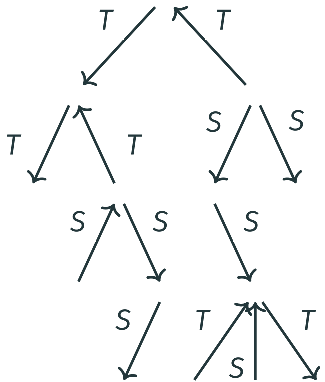
I:



Q:

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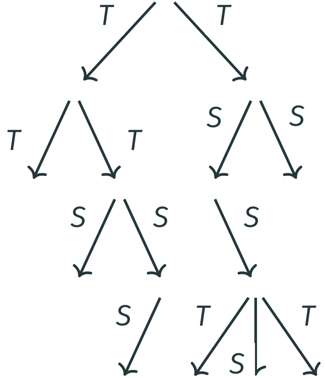
$\mathcal{Q}$ = *one-way paths*, $\mathcal{I}$ = *downwards trees*

- $\mathcal{Q}$ = one-way paths (1WP), $\mathcal{I}$ = **downwards trees** (DWT)

Q:

I:



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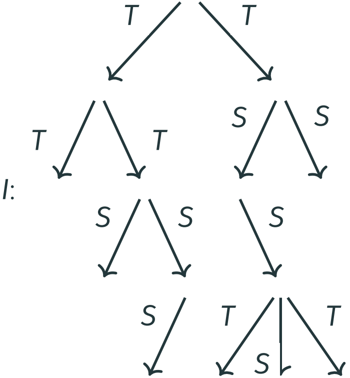

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- Global orientation of the instance also has an impact!

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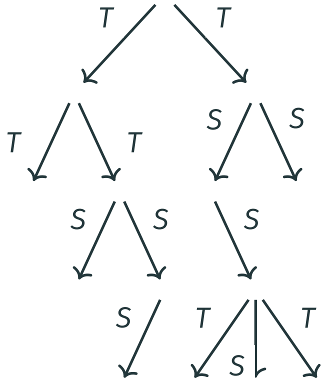
I:



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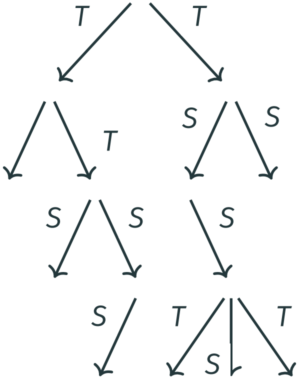
I:



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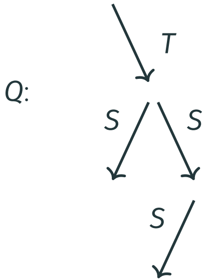

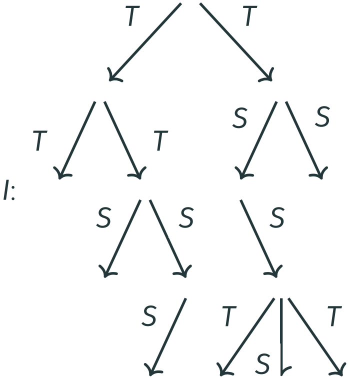

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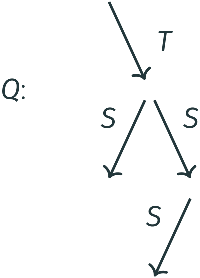
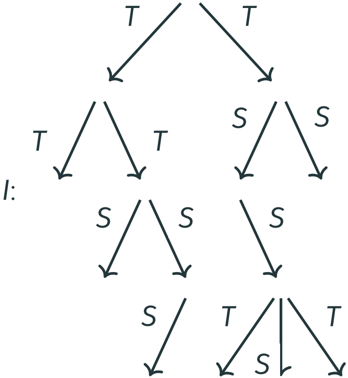
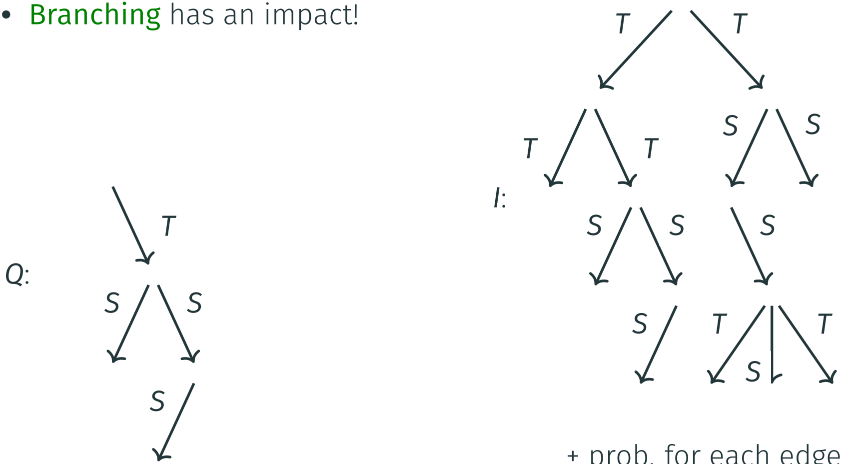
- **#P-hard**





+ prob. for each edge

- $\mathcal{Q}$ = **downwards trees** (DWT), $\mathcal{I}$ = downwards trees
- **#P-hard**
- Branching has an impact!



+ prob. for each edge

Our graph classes

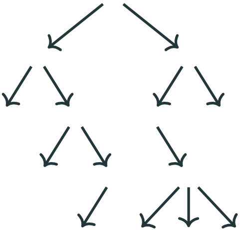
1WP

$\xrightarrow{R} \xrightarrow{S} \xrightarrow{S} \xrightarrow{T}$

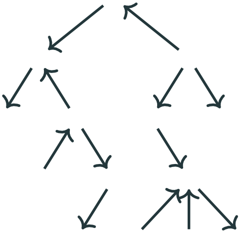
2WP

$\xrightarrow{R} \xleftarrow{S} \xleftarrow{S} \xrightarrow{T} \xleftarrow{R}$

DWT



PT



1WP $\subseteq$ 2WP $\subseteq$ PT

1WP $\subseteq$ DWT $\subseteq$ PT

PT $\subseteq$ Connected $\subseteq$ All

Results

$\downarrow Q$ $I \rightarrow$	1WP	2WP	DWT	PT	Connected
1WP					
2WP					
DWT		PTIME			
PT					#P-hard
Connected					

$\geqslant 2$ labels

Results

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1WP					
2WP					
DWT		PTIME			
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$\geqslant$ 2 labels

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2WP					
DWT		PTIME			
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Connected					

No labels

Results

$\downarrow Q$ $\quad I\rightarrow$	1WP	2WP	DWT	PT	Connected
1WP			•	•	
2WP			•		
DWT		PTIME	•		
PT					#P-hard
Connected		•			

$\geqslant 2$ labels

$\downarrow Q$ $\quad I\rightarrow$	1WP	2WP	DWT	PT	Connected
1WP					•
2WP				•	
DWT		PTIME		•	
PT					#P-hard
Connected		•	•		

No labels

Results

$\geqslant 2$ labels

$\downarrow Q$ $I \rightarrow$	1WP	2WP	DWT	PT	Connected
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2WP					
DWT		PTIME			
PT					#P-hard
Connected					

No labels

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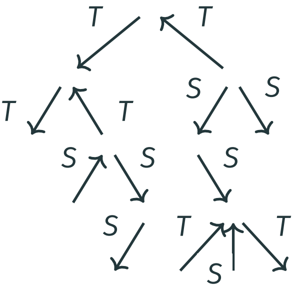
1WP					
2WP					
DWT		PTIME			
PT					#P-hard
Connected					

Reduction for $\mathcal{Q}$ = *one-way paths*, $\mathcal{I}$ = *polytrees*

Q: $\xrightarrow{T}\ \xrightarrow{S}\ \xrightarrow{S}\ \xrightarrow{S}\ \xrightarrow{T}$

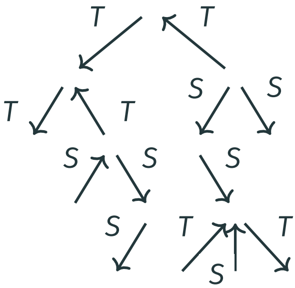
I:



+ prob. for each edge

Reduction for $\mathcal{Q}$ = *one-way paths*, $\mathcal{I}$ = *polytrees*

Q: 

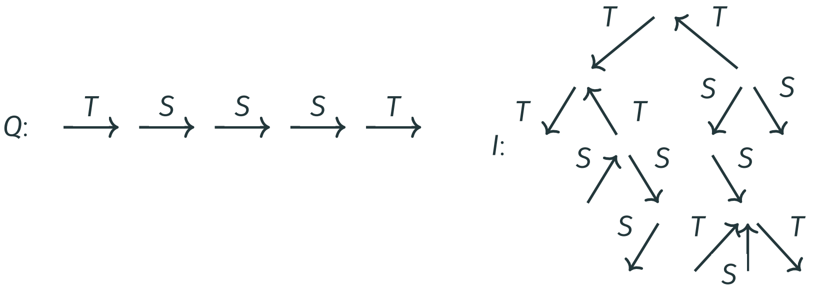
I: 

+ prob. for each edge

Reduction from **#P-hard** problem `#PP2DNF`:

- INPUT: Boolean formula $\varphi = \bigvee_{j=1\ldots m}(X_{x_j} \wedge Y_{y_j})$ on variables $\{X_1, \ldots, X_{n_1}\} \sqcup \{Y_1, \ldots, Y_{n_2}\}$

Reduction for $\mathcal{Q}$ = *one-way paths*, $\mathcal{I}$ = *polytrees*



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- OUTPUT: number of satisfying assignments of φ

Reduction for $\mathcal{Q}$ = *one-way paths*, $\mathcal{I}$ = *polytrees*

$\varphi = X_1 Y_2 \lor X_1 Y_1 \lor X_2 Y_2$

Reduction for $\mathcal{Q}$ = *one-way paths*, $\mathcal{I}$ = *polytrees*

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I:

Q:

Reduction for $\mathcal{Q}$ = *one-way paths*, $\mathcal{I}$ = *polytrees*

$\varphi = X_1 Y_2 \vee X_1 Y_1 \vee X_2 Y_2$

I:

Q:

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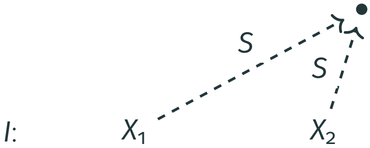
$\varphi = X_1 Y_2 \lor X_1 Y_1 \lor X_2 Y_2$



Q:

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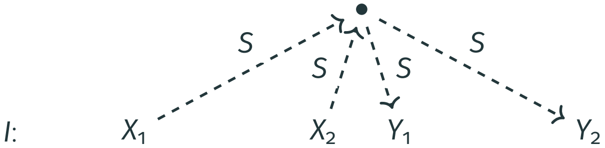

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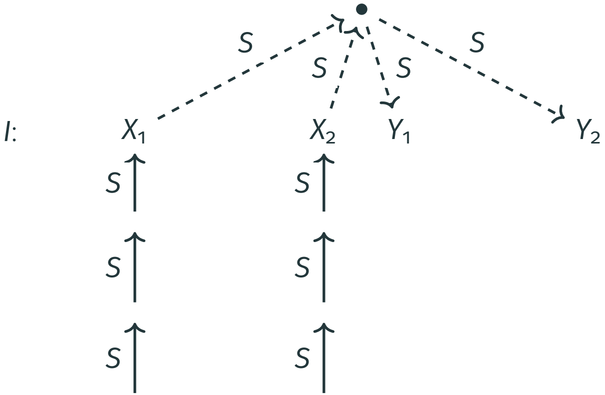

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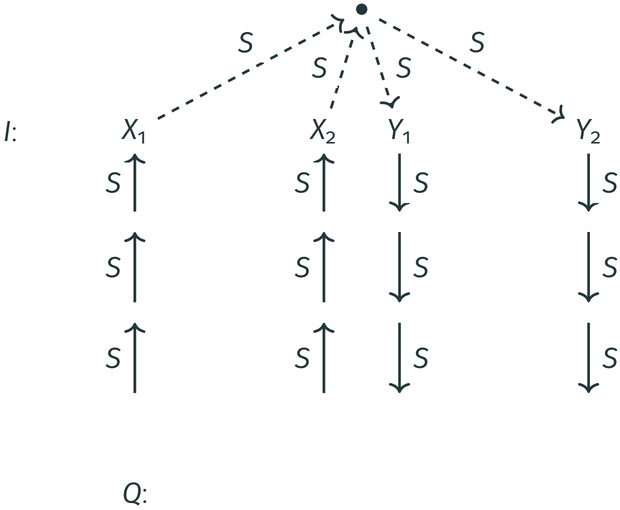

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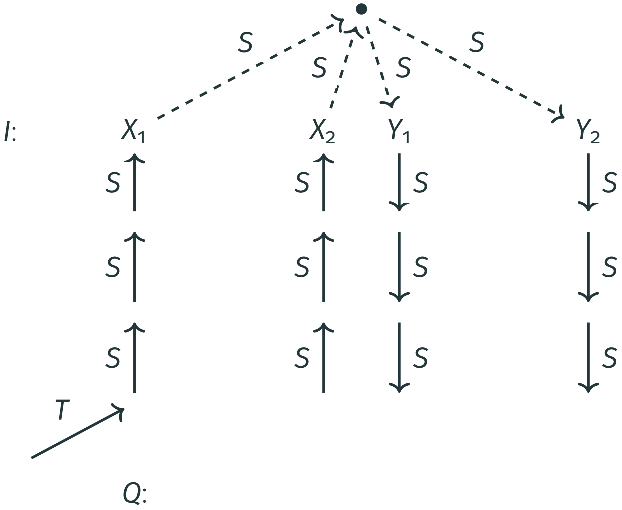

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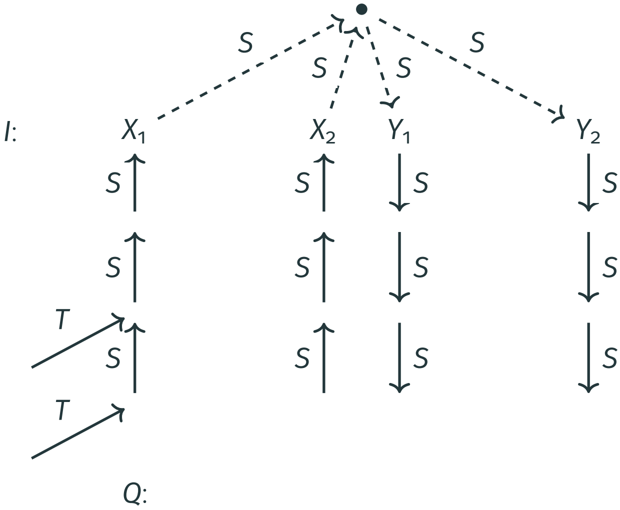

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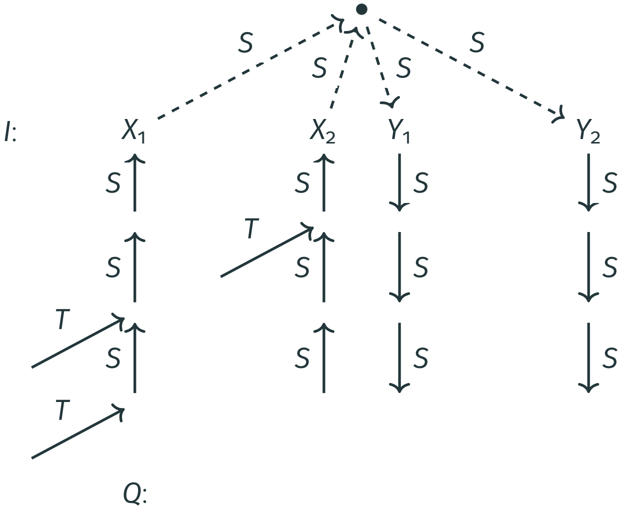

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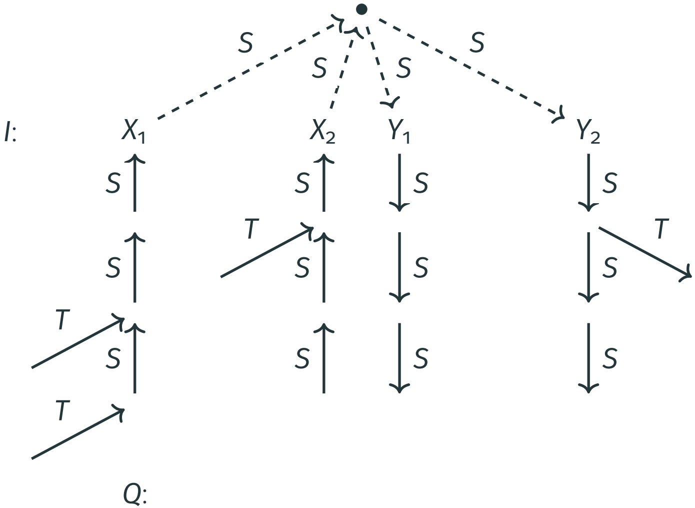

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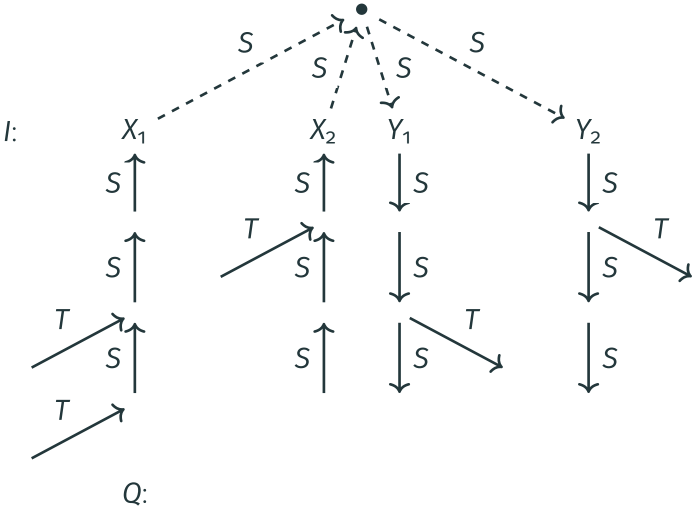

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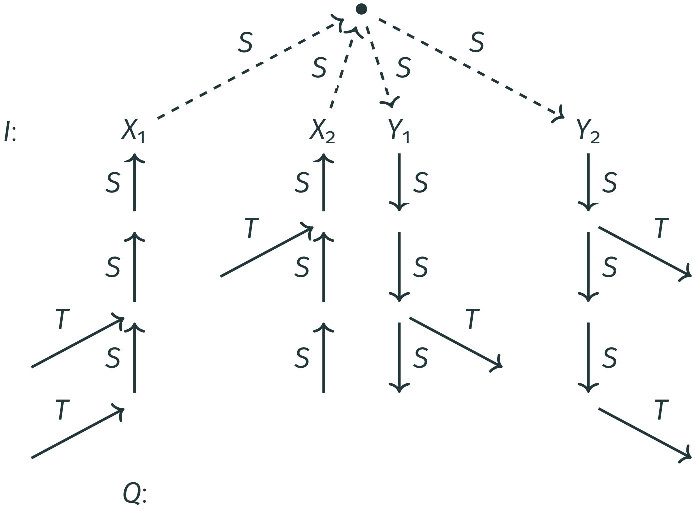

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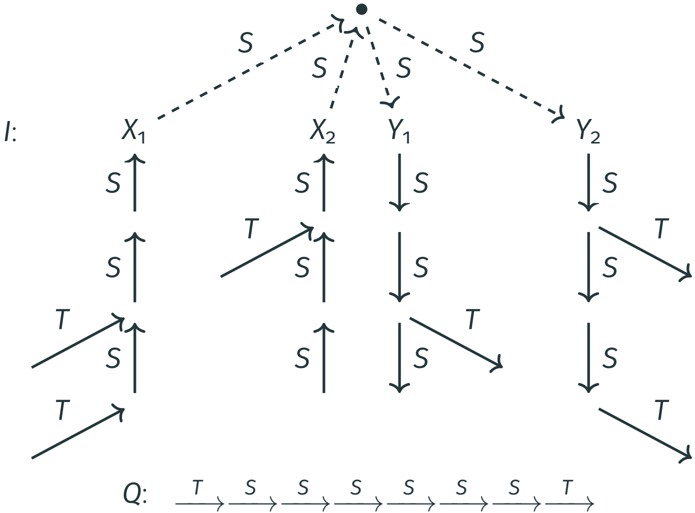

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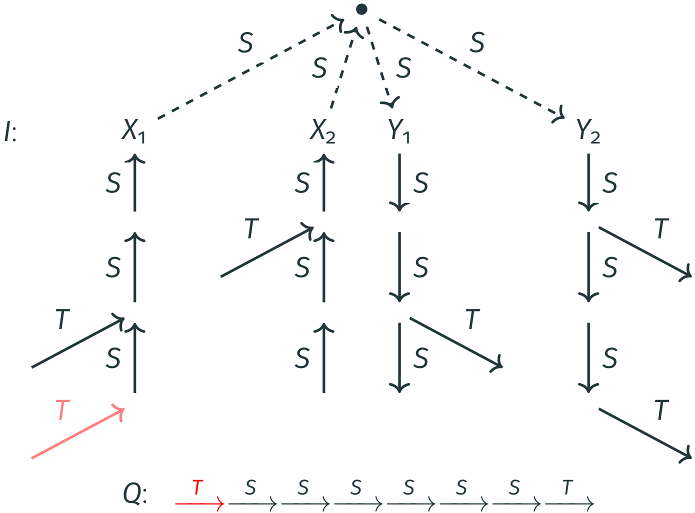

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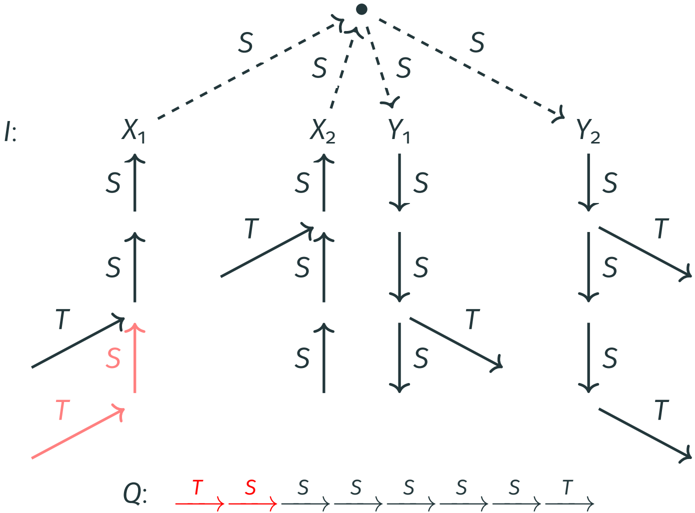

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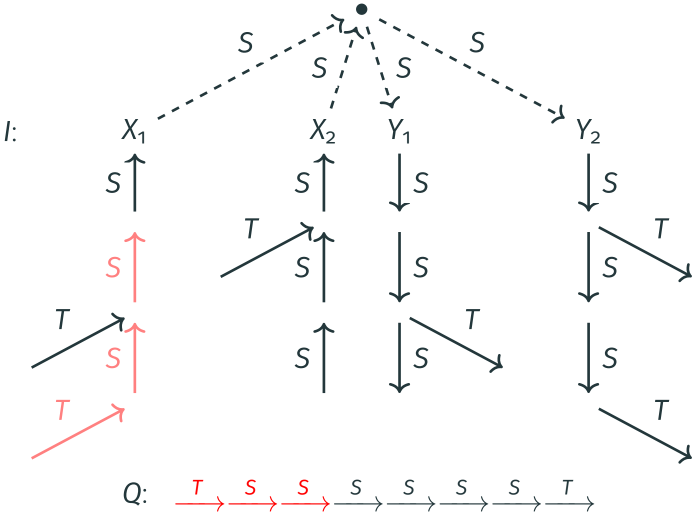

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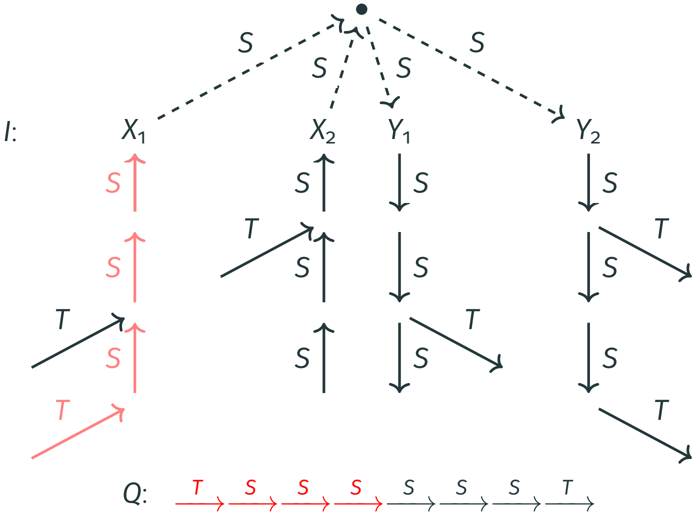

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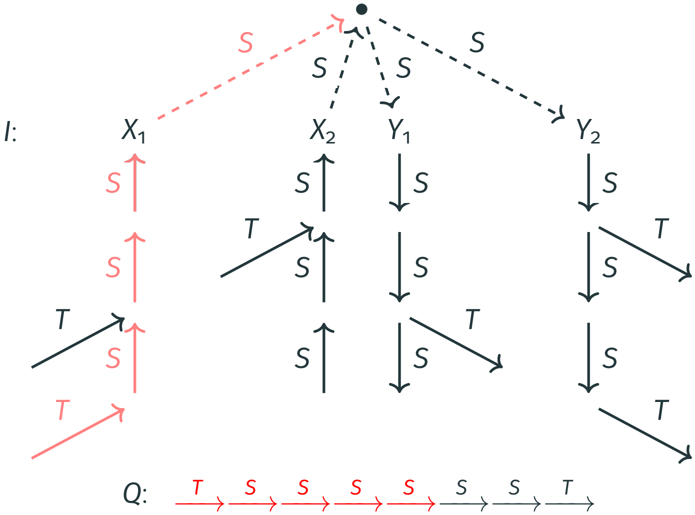

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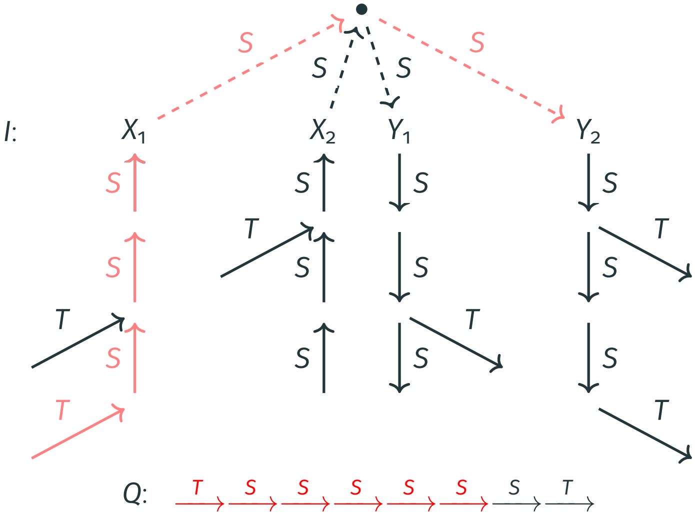

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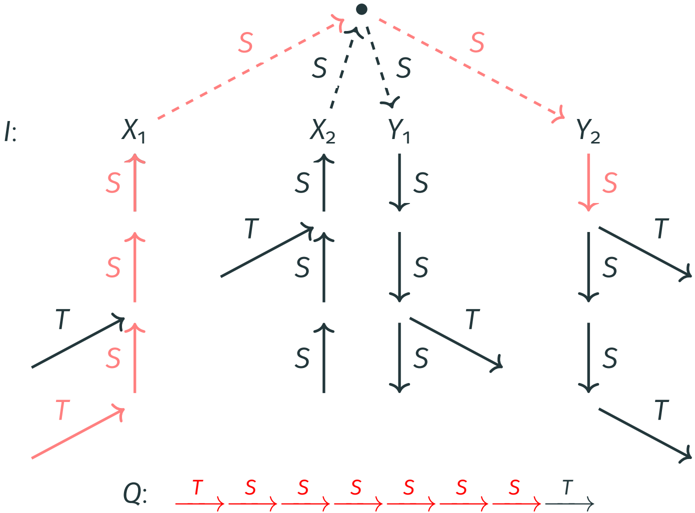

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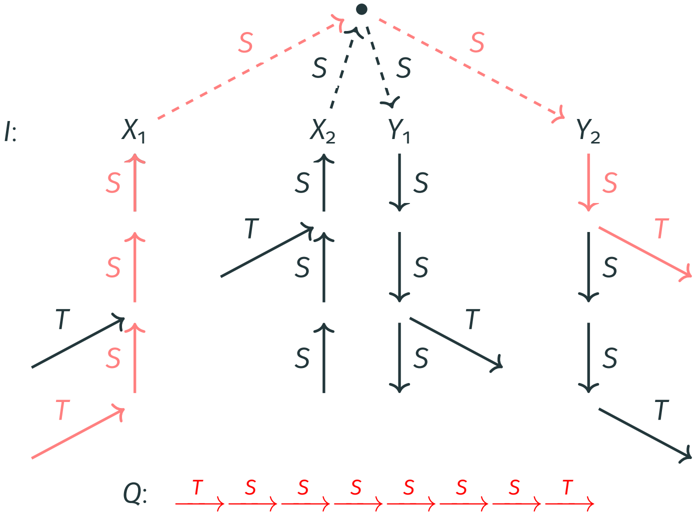

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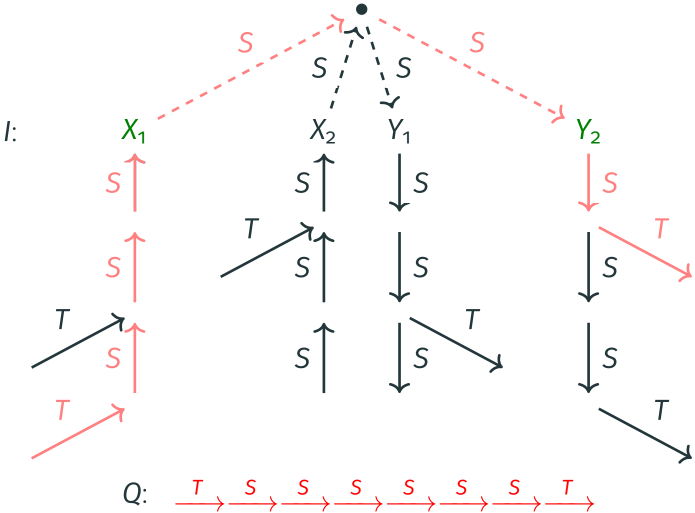

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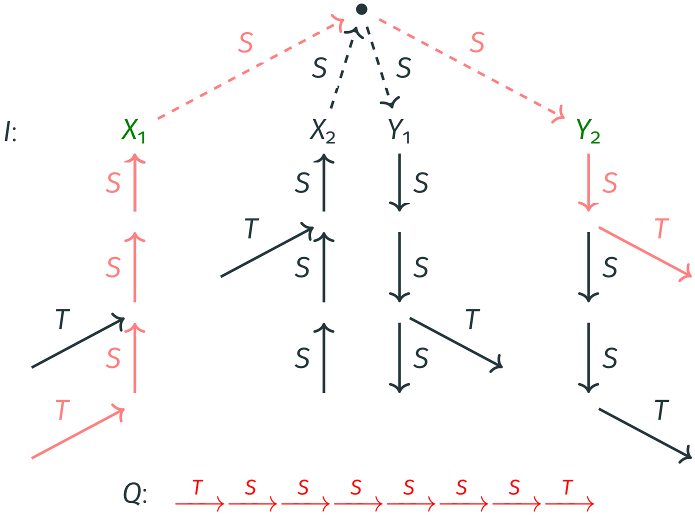

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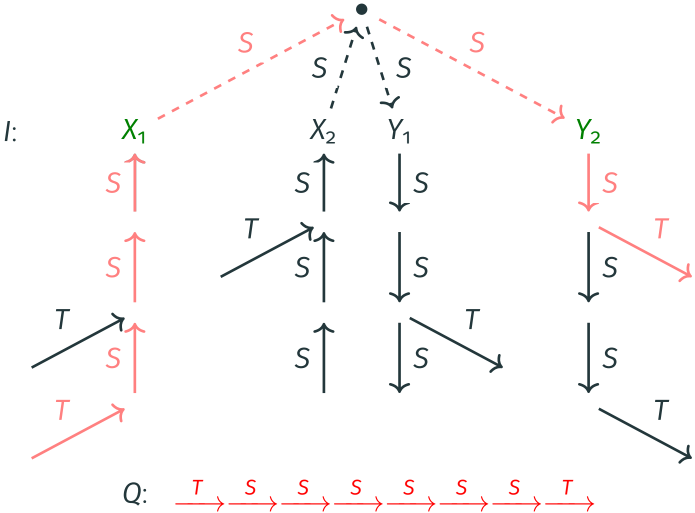
$\#\varphi = \Pr((I, \pi) \models Q) \times 2^{|\mathrm{vars}(\varphi)|}$



Disconnected graphs

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With labels, PQE of $\bigsqcup$1WP on 1WP is already **#P-hard**!

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Thanks for your attention!