Graph

Charles Lin

Agenda

- **Graph Representation**
- **DFS**
- **BFS**
- **Dijkstra**
- **A* Search**
- **Bellman-Ford**
- **Floyd-Warshall**
- **Iterative? Non-iterative?**
- **MST**
- **Flow – Edmond-Karp**

Graph Representation

- **Adjacency Matrix**
  - **bool way[100][100];**
  - **cin >> i >> j;**
  - **way[i][j] = true;**

Graph Representation

- **Adjacency Linked List**
  - **vector<int> v[100];**
  - **cin >> i >> j;**
  - **v[i].push_back(j);**

Graph Representation

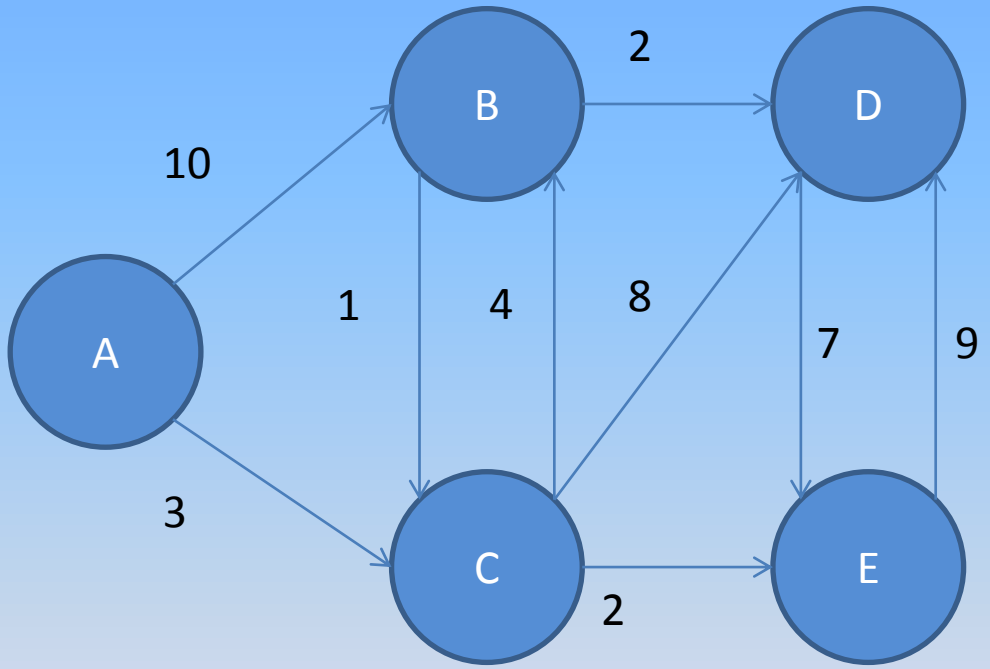
- **Edge List**

```
struct Edge {
    int Start_Vertex, End_Vertex;
} edge[10000];
cin >> edge[i].Start_Vertex  >>
edge[i].End_Vertex;
```

Graph Representation

	Adjacency Matrix	Adjacency Linked List	Edge List
Memory Storage	**$O(V^2)$**	**O(V+E)**	**O(V+E)**
Check whether (*u*,*v*) is an edge	**O(1)**	**O(deg(u))**	**O(deg(u))**
Find all adjacent vertices of a vertex *u*	**O(V)**	**O(deg(u))**	**O(deg(u))**
deg(u): the number of edges connecting vertex *u*			

Graph Theory



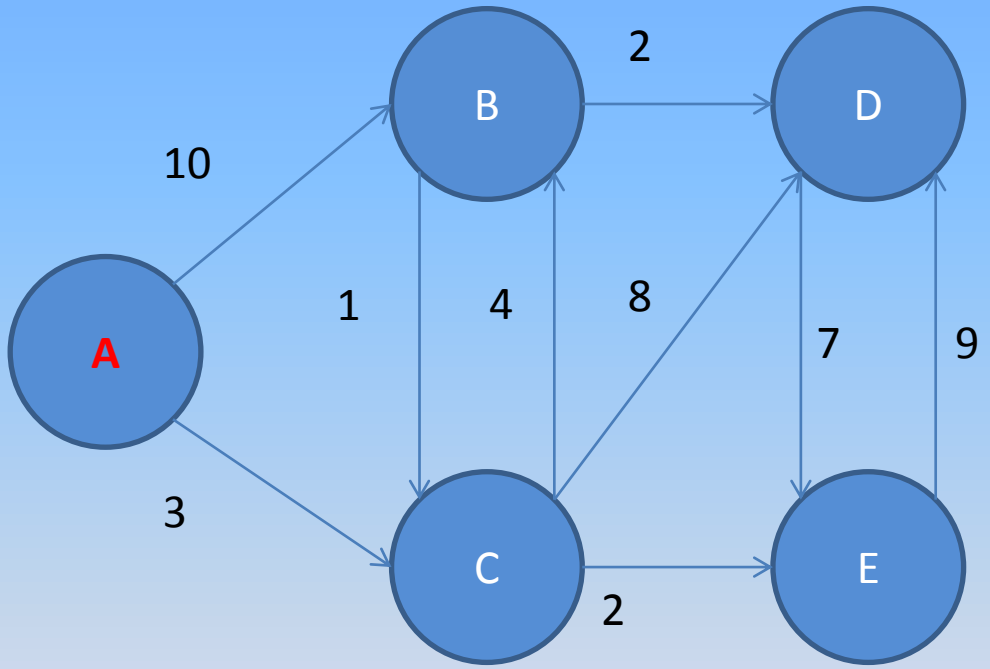
Graph Theory

- **Mission: To go from Point A to Point E**
- **How?**

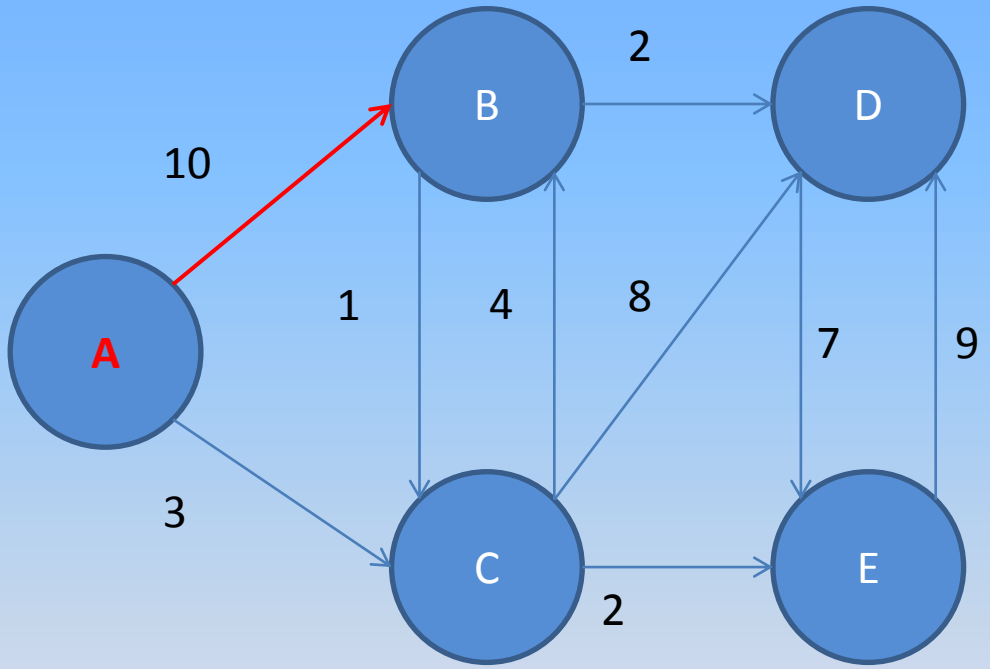
Depth First Search (DFS)

- **Structure to use: Stack**
- **See “The House of Santa Claus”**

Depth First Search (DFS)



Depth First Search (DFS)



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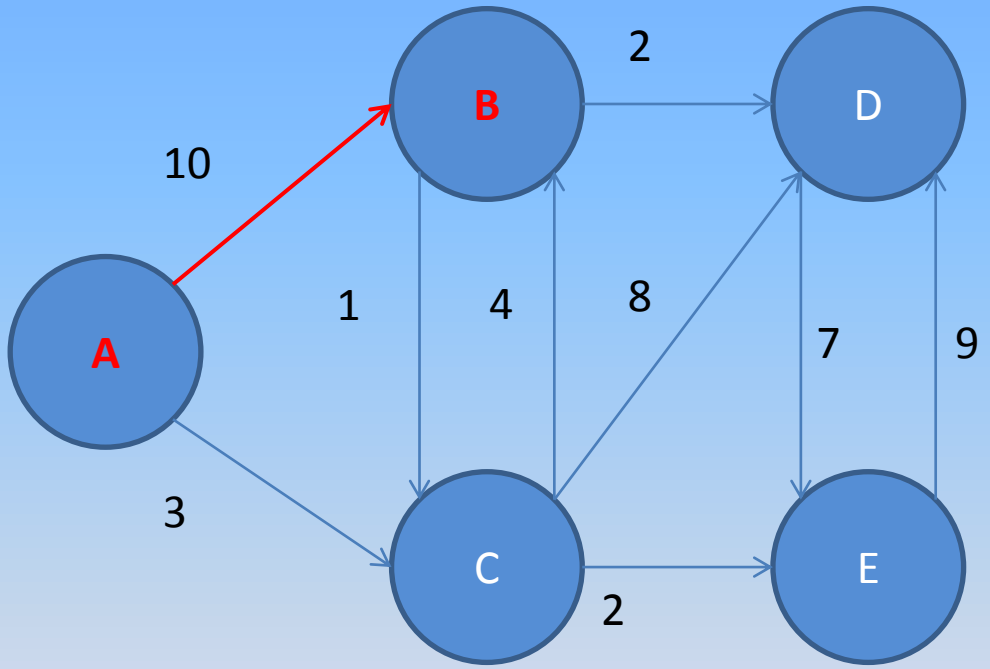
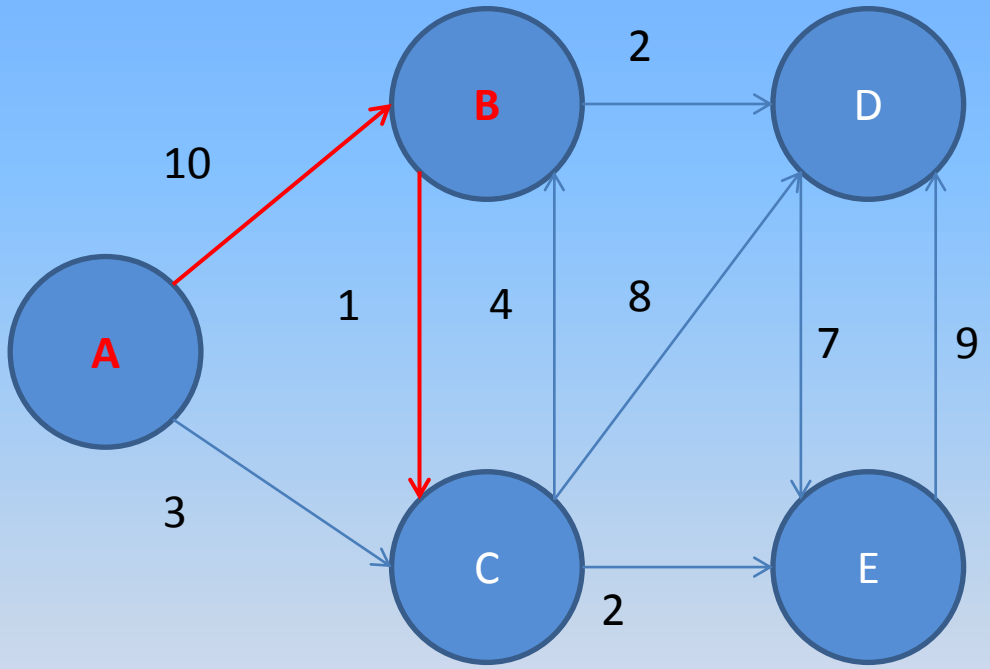
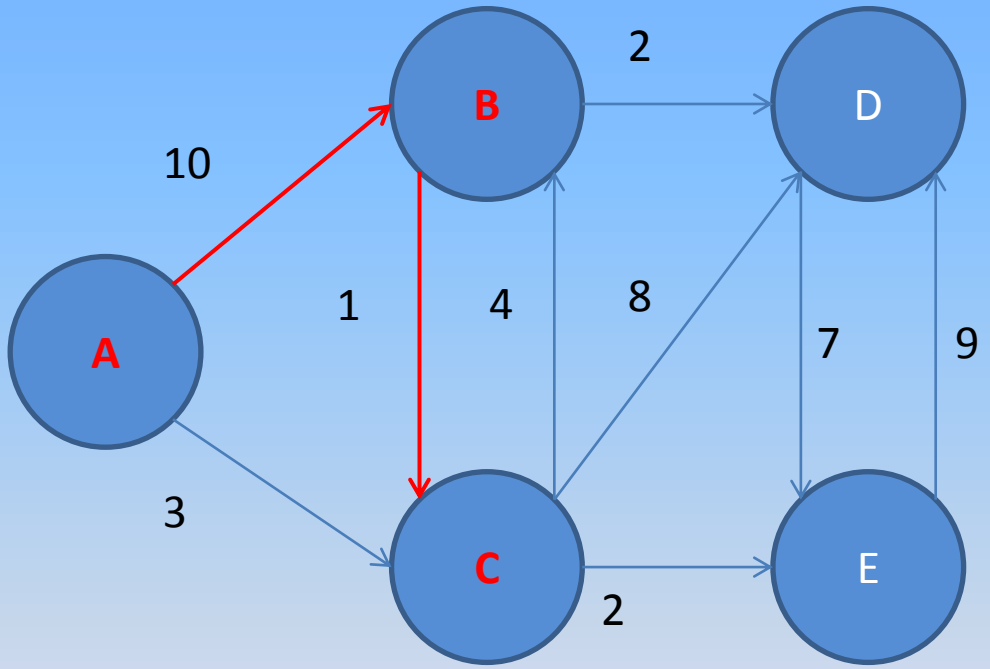


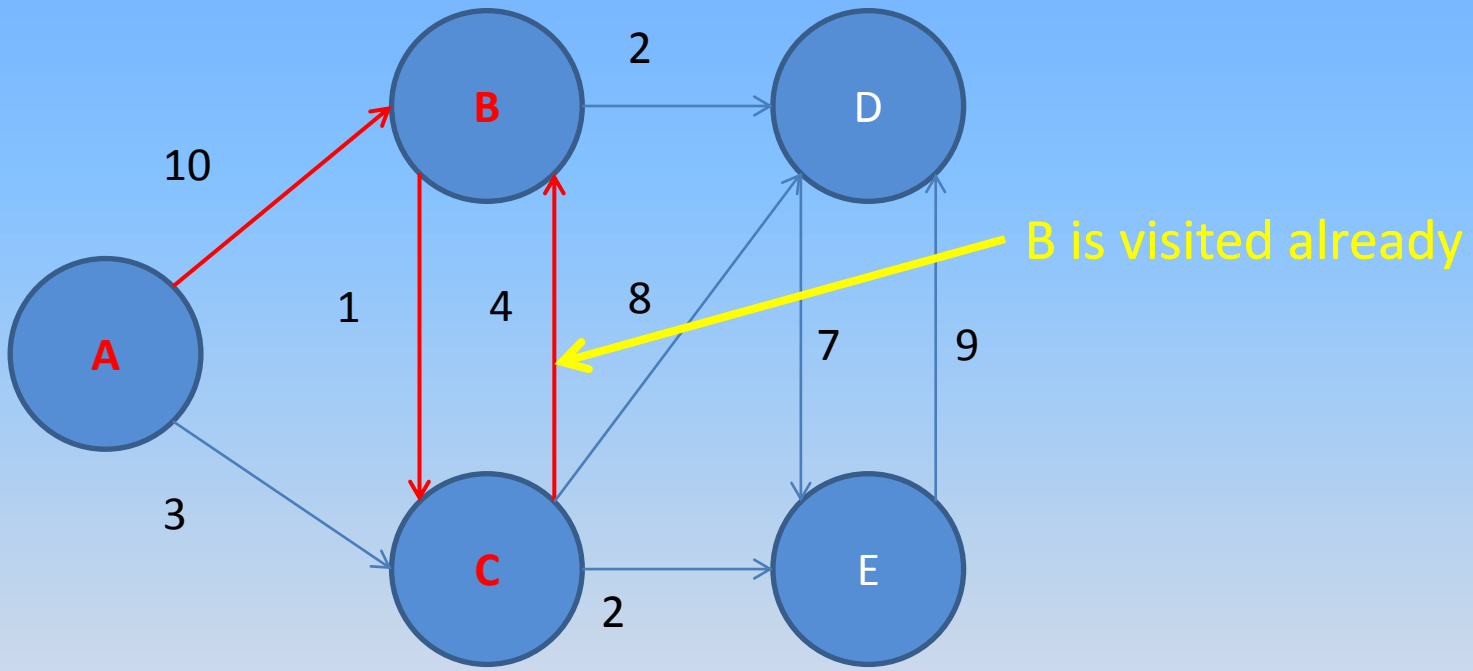
Depth First Search (DFS)



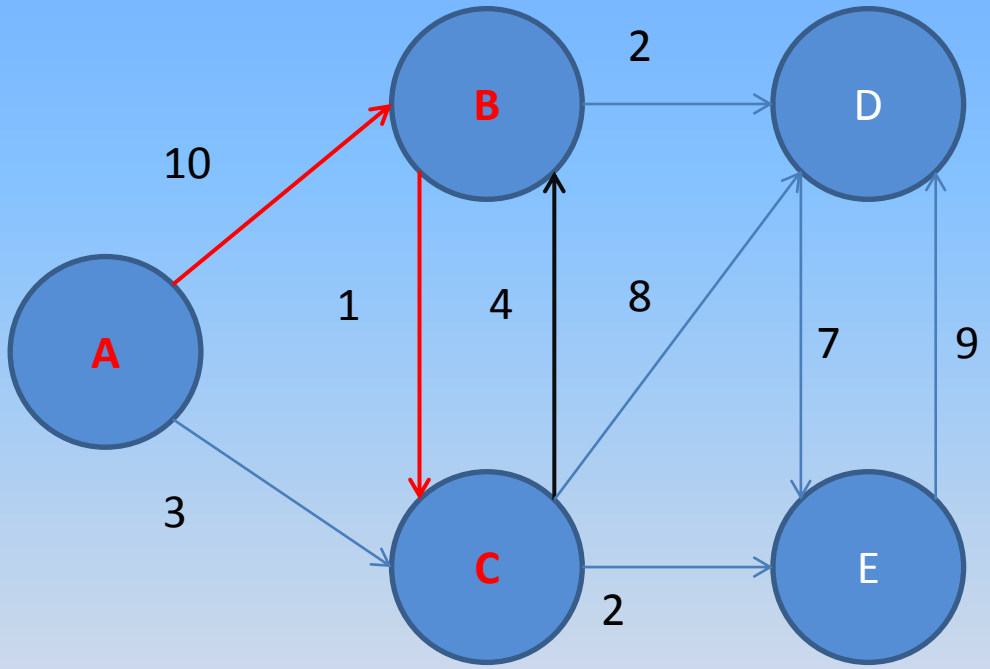
Depth First Search (DFS)



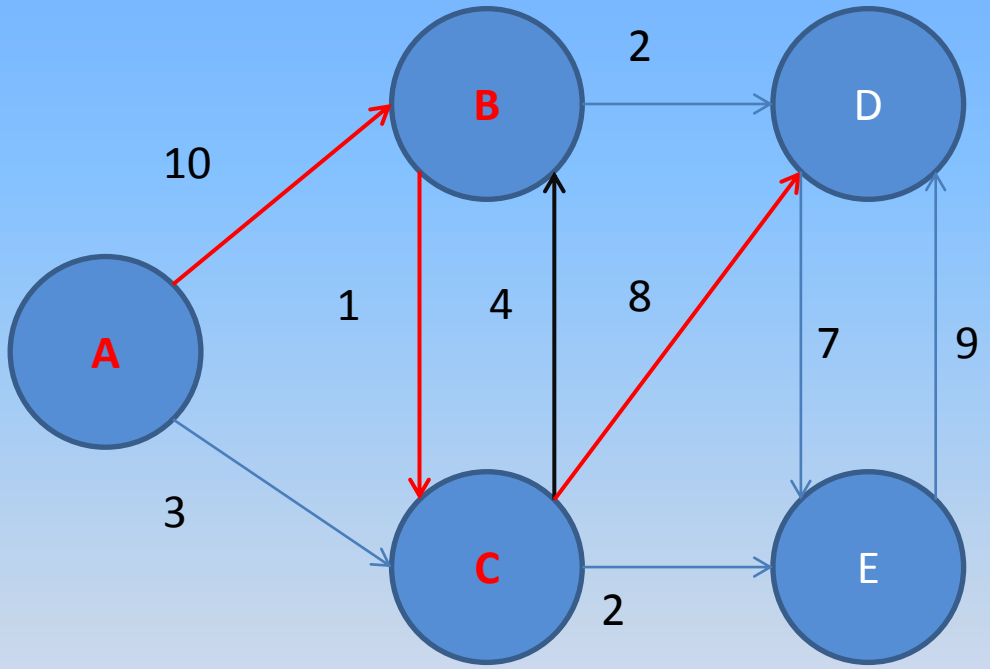
Depth First Search (DFS)



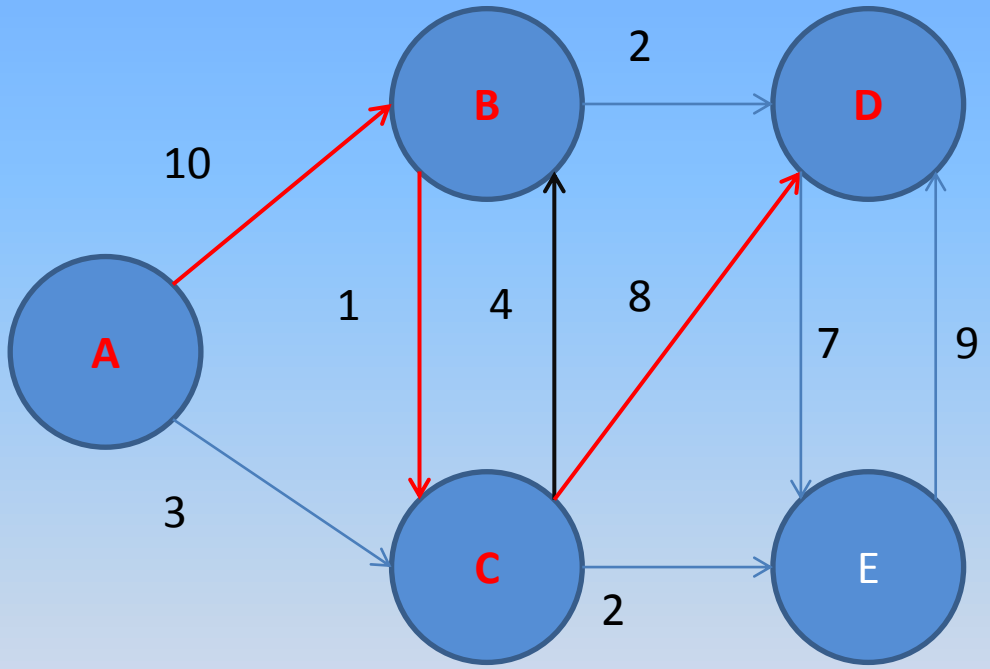
Depth First Search (DFS)



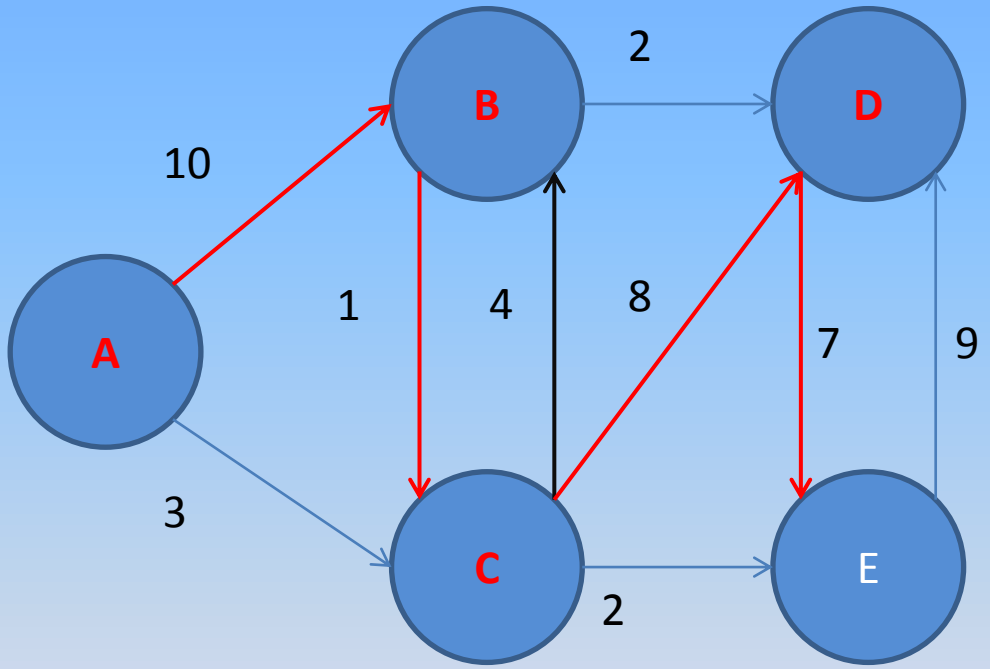
Depth First Search (DFS)



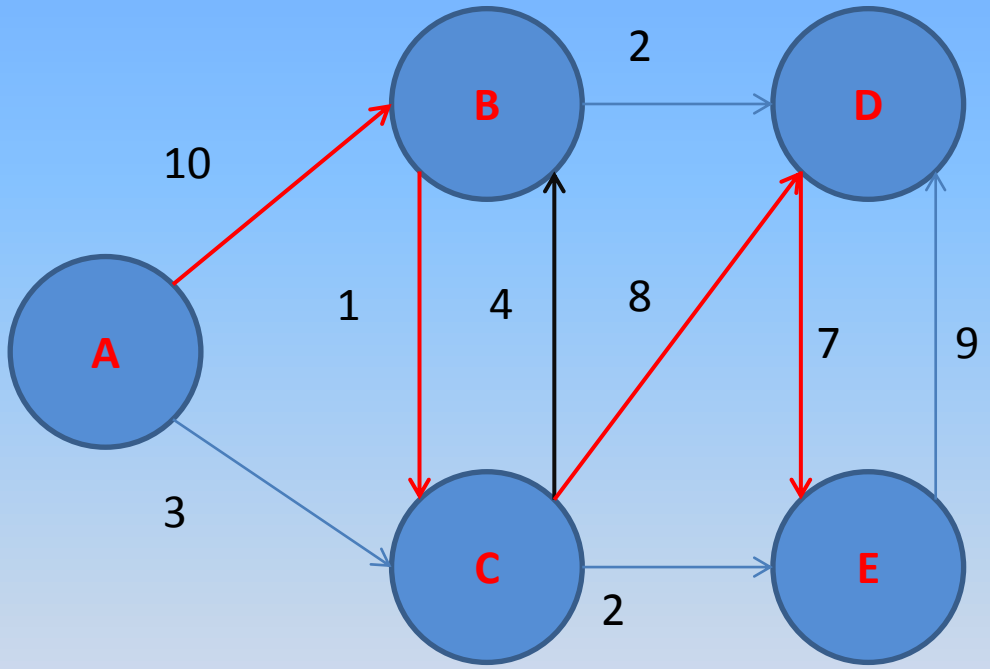
Depth First Search (DFS)



Depth First Search (DFS)



Depth First Search (DFS)



We find a path from Point A to Point E, but ……

Depth First Search (DFS)

```
stack<int> s;
bool Visited[MAX];

void DFS(int u) {
   s.push(u);
   if (u == GOAL) {
      for (int x = 0; x < s.size(); x++)
         printf("%d ", s[x]);
      return ;
   }
   for (int x = 0; x < MAX; x++)
      if (!Visited[x]) {
         Visited[x] = true;
         DFS(x);
         Visited[x] = false;
      }
   s.pop();
}
```

```
int main() {
   memset(Visited, 0, sizeof(Visited));
   // Input Handling (GOAL = ?)
   DFS(0);
}
```

Very Important
It needs to be restored for other points to traverse, or the path may not be found

Depth First Search (DFS)

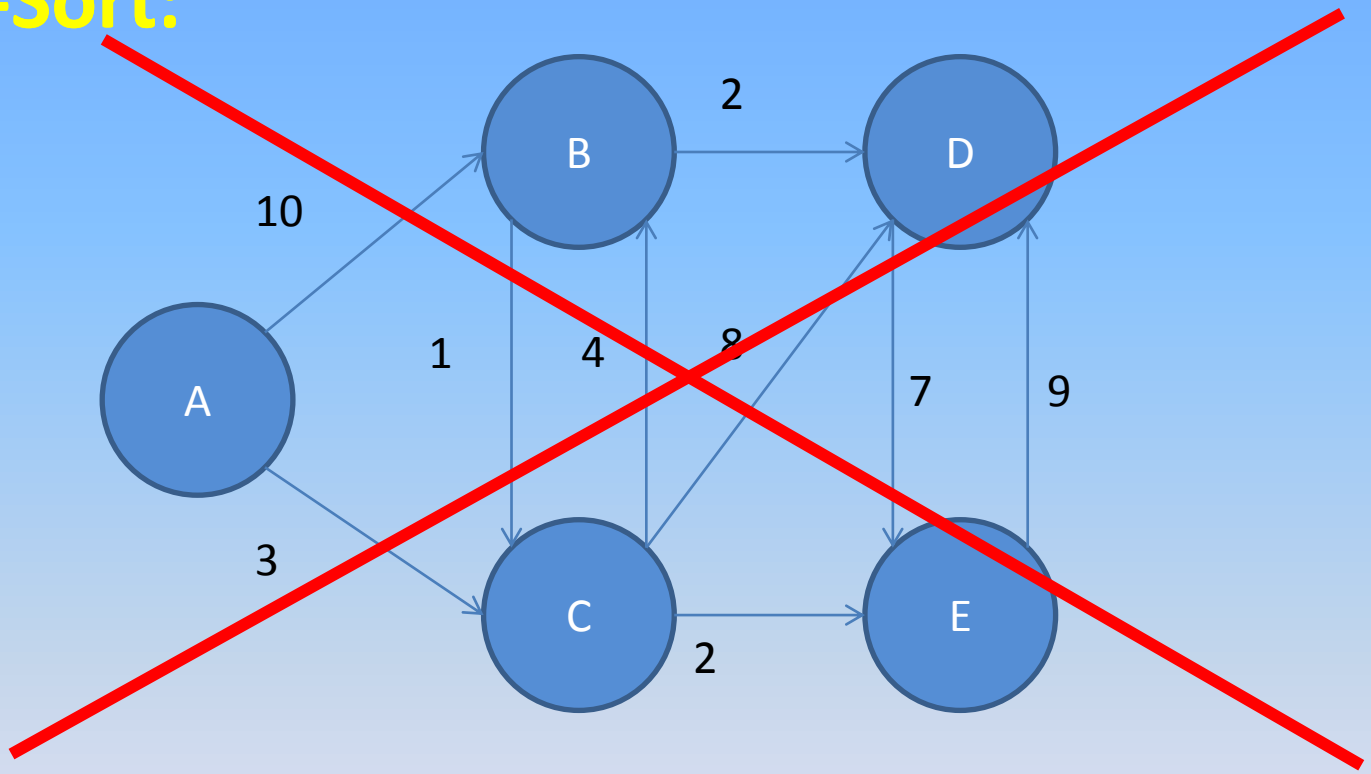
- **Usage:**
  - **Finding a path from start to destination (NOT RECOMMENDED – TLE)**
  - **Topological Sort (T-Sort)**
  - **Strongly-Connected Component (SCC)**

Depth First Search (DFS)

- **Topological Sort**
  - **Directed Acyclic Graph (DAG)**
  - **Find the order of nodes such that for each node, every parent is before that node on the list**
  - **Dump Method: Check for root of residual graph (Do it n times)**
  - **Better Method: Reverse of finishing time**

Depth First Search (DFS)

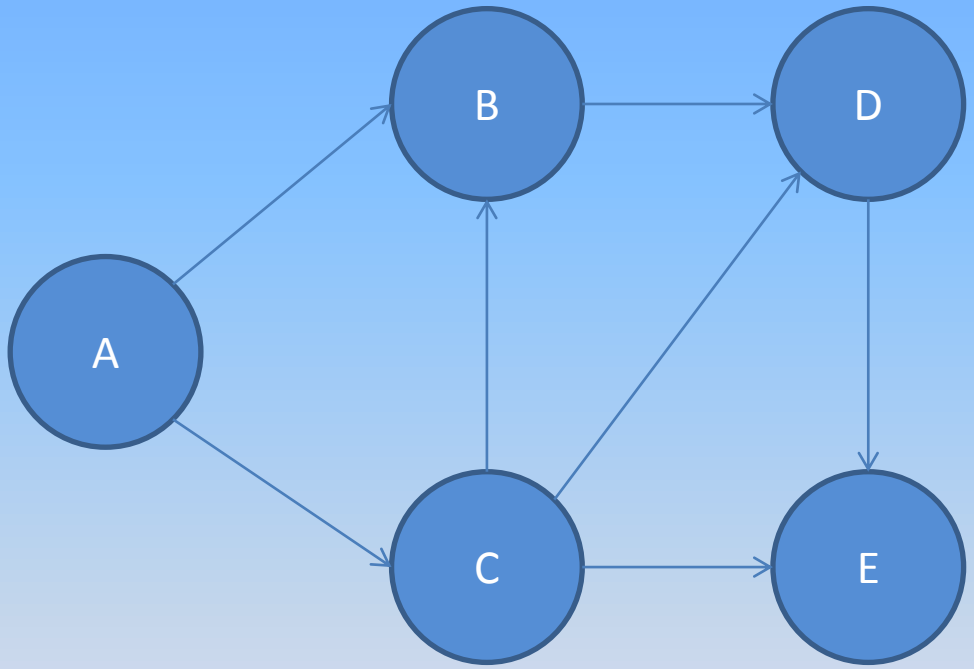
T-Sort:



Cycle Exists !!!

Depth First Search (DFS)

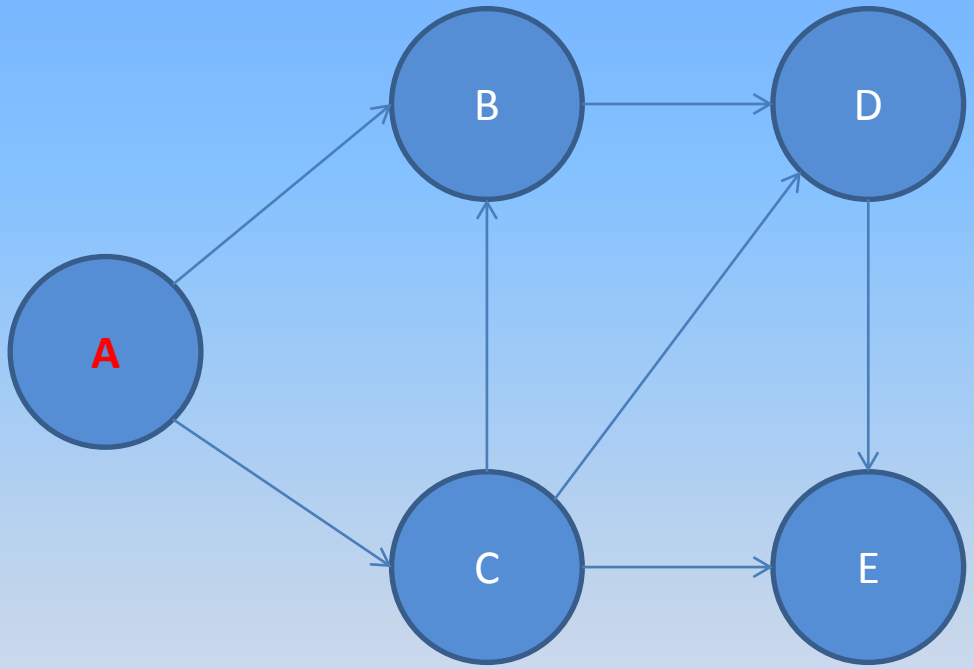
T-Sort:



OK

Depth First Search (DFS)

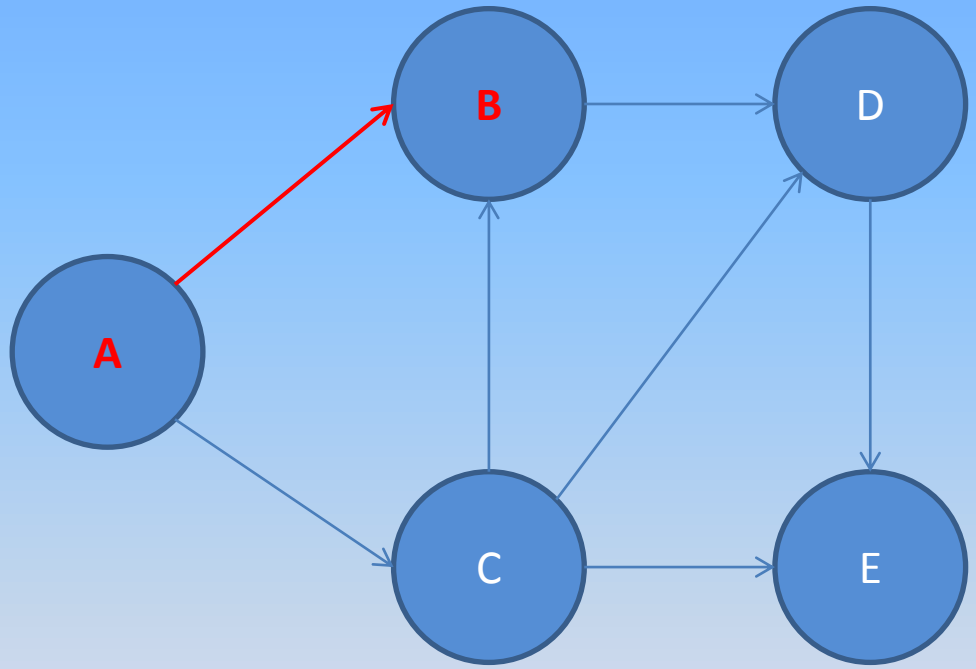
T-Sort:



Things in output stack: NONE

Depth First Search (DFS)

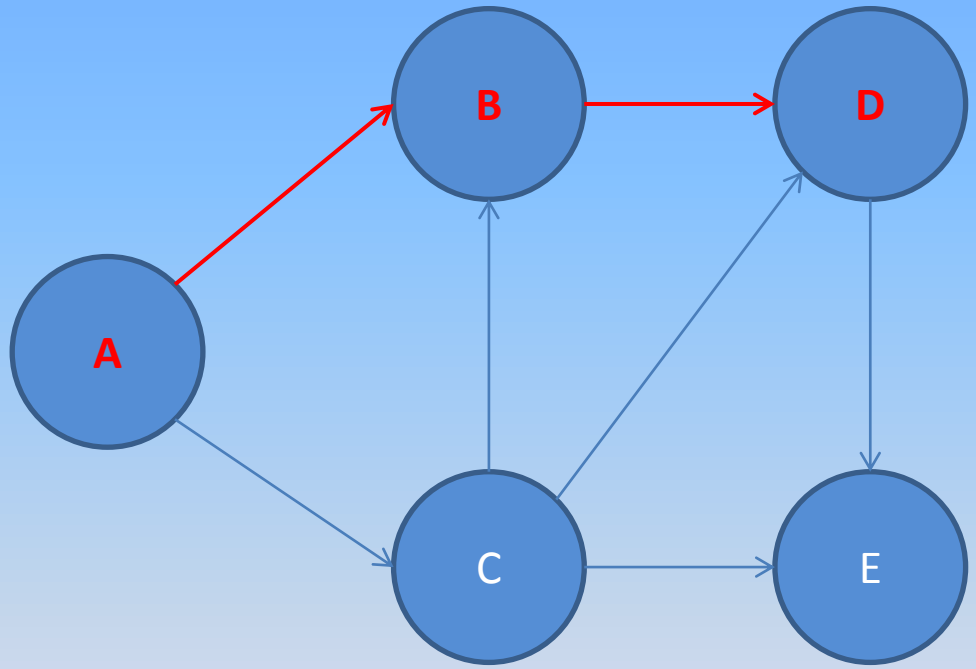
T-Sort:



Things in output stack: NONE

Depth First Search (DFS)

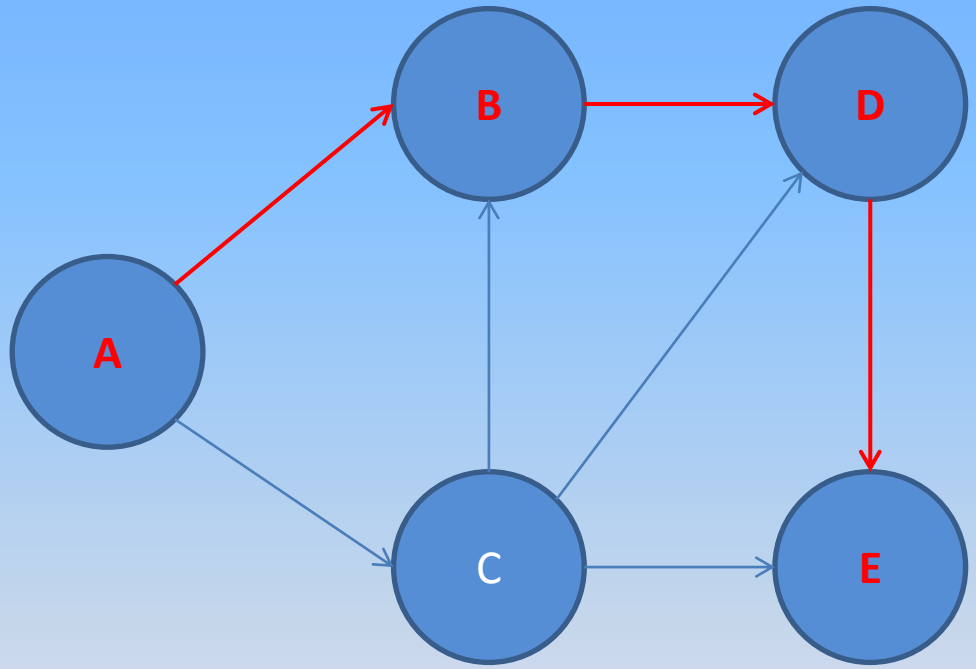
T-Sort:



Things in output stack: NONE

Depth First Search (DFS)

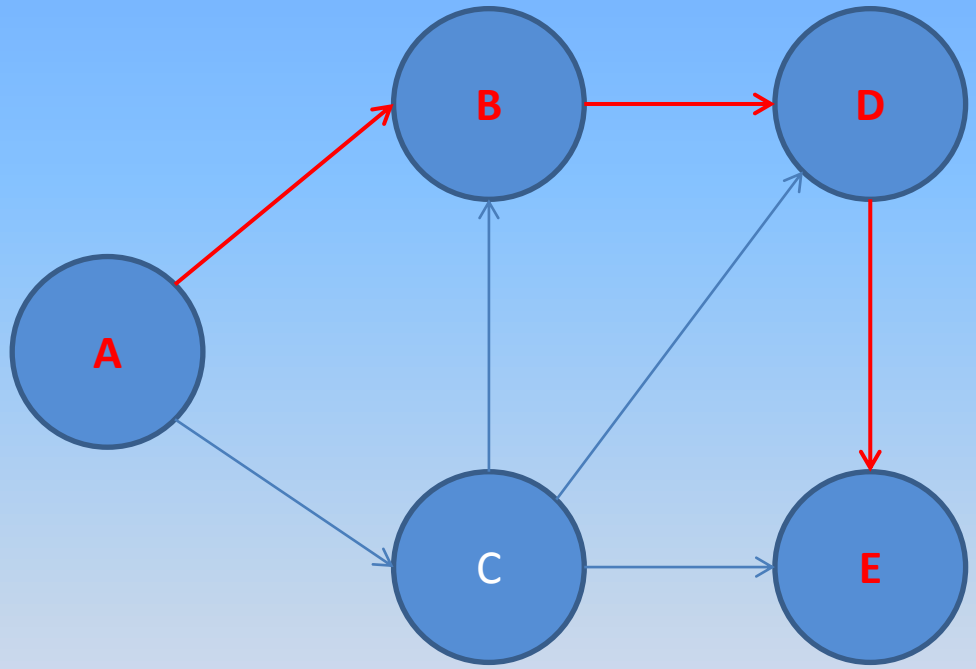
T-Sort:



Things in output stack: NONE

Depth First Search (DFS)

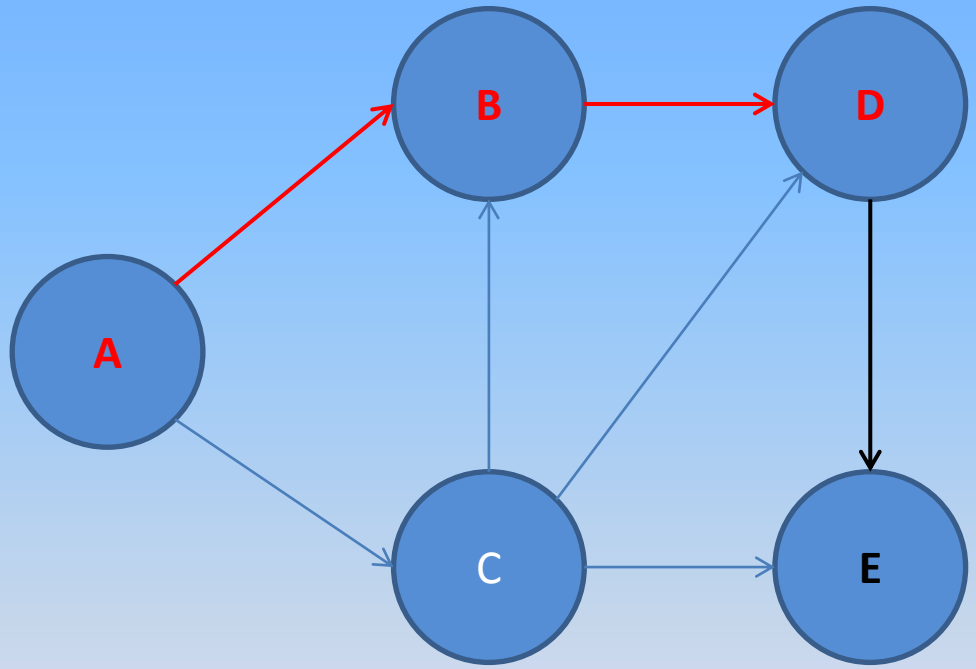
T-Sort:



Things in output stack: NONE

Depth First Search (DFS)

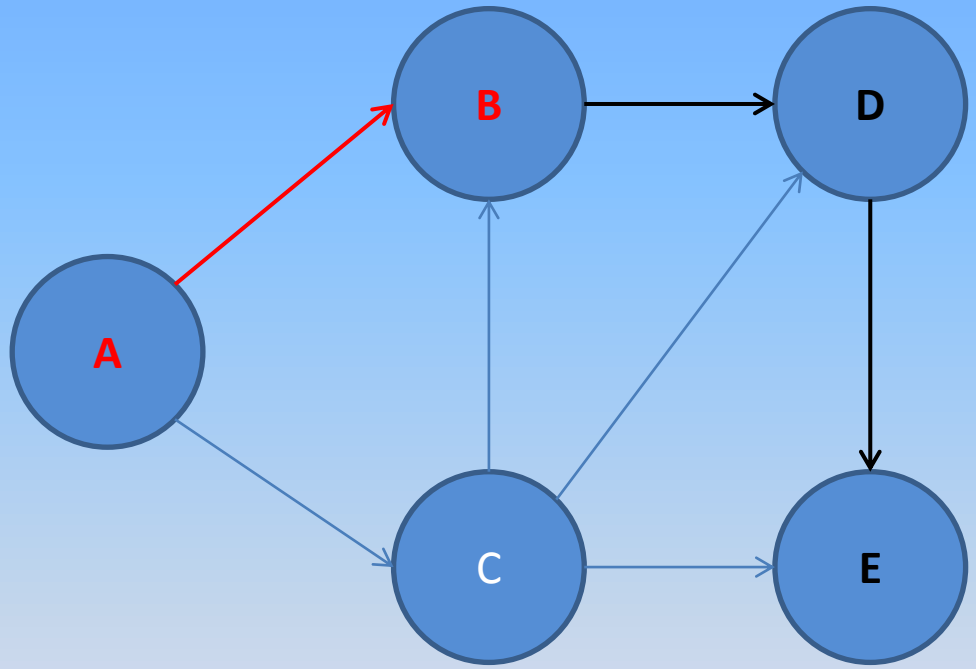
T-Sort:



Things in output stack: E

Depth First Search (DFS)

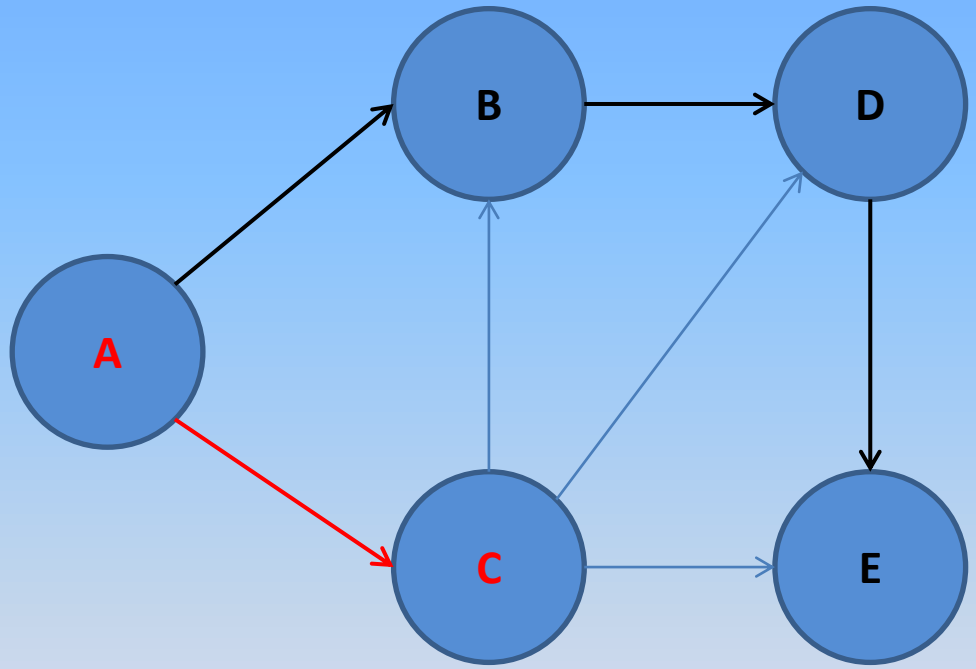
T-Sort:



Things in output stack: E, D

Depth First Search (DFS)

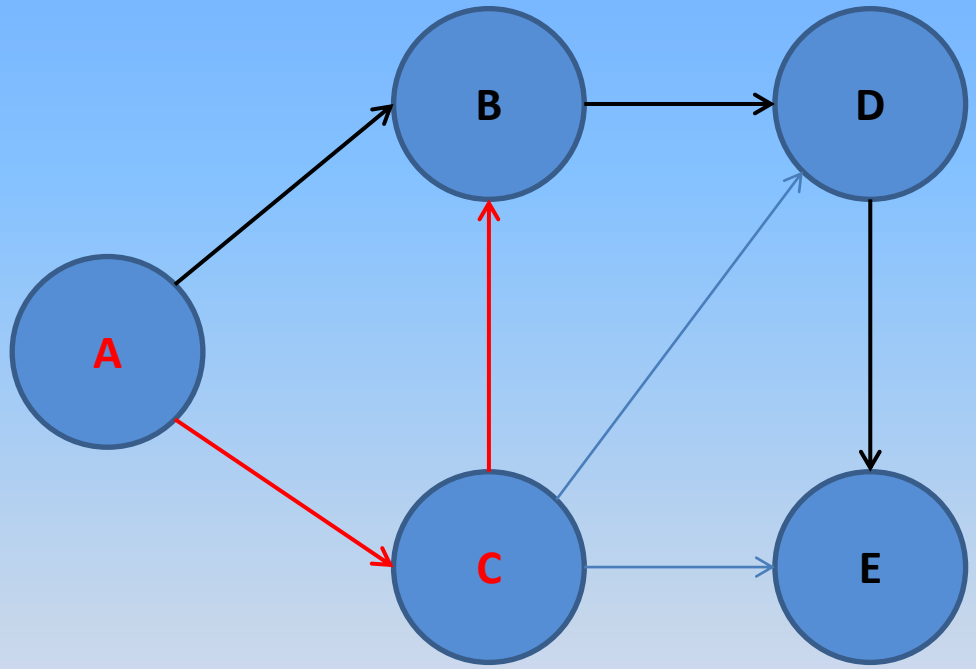
T-Sort:



Things in output stack: E, D, B

Depth First Search (DFS)

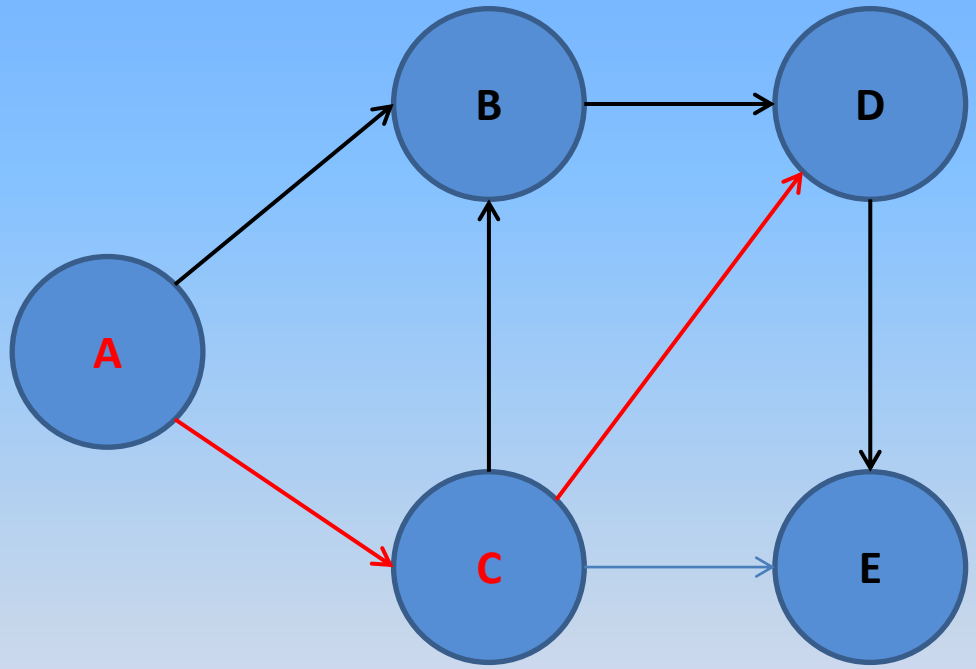
T-Sort:



Things in output stack: E, D, B

Depth First Search (DFS)

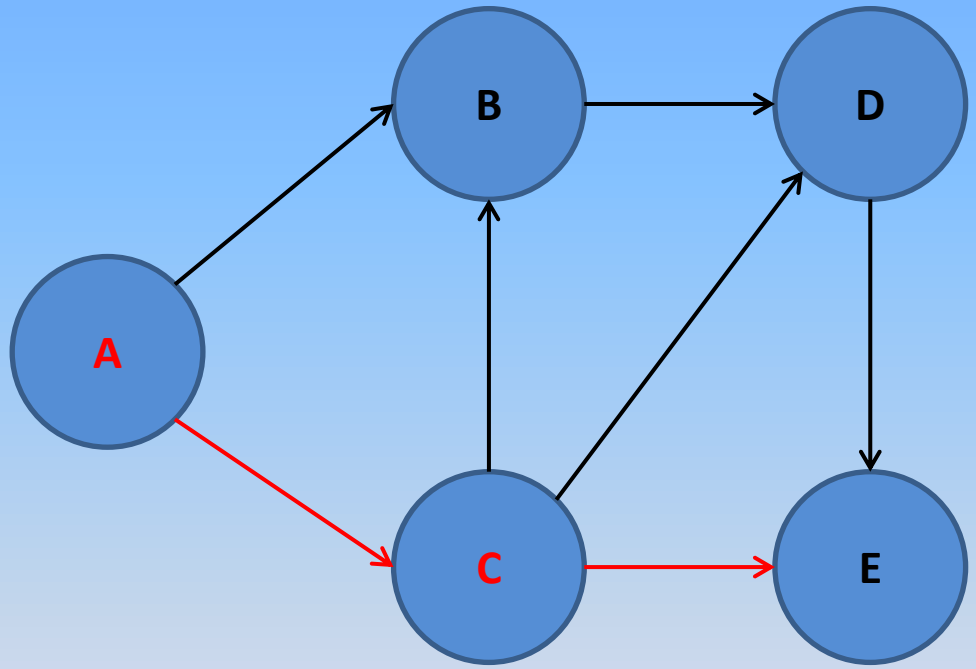
T-Sort:



Things in output stack: E, D, B

Depth First Search (DFS)

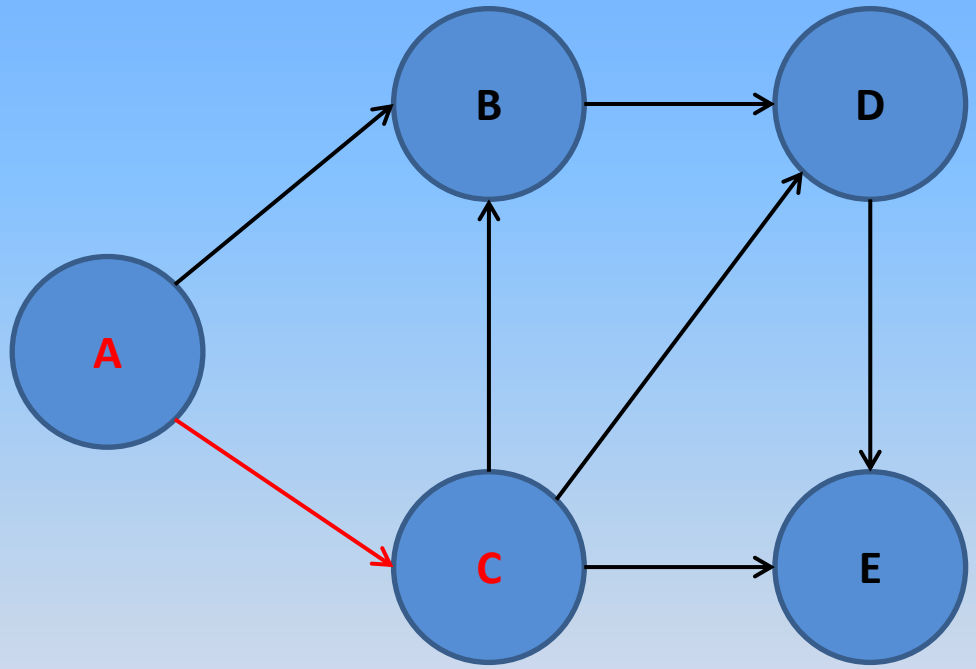
T-Sort:



Things in output stack: E, D, B

Depth First Search (DFS)

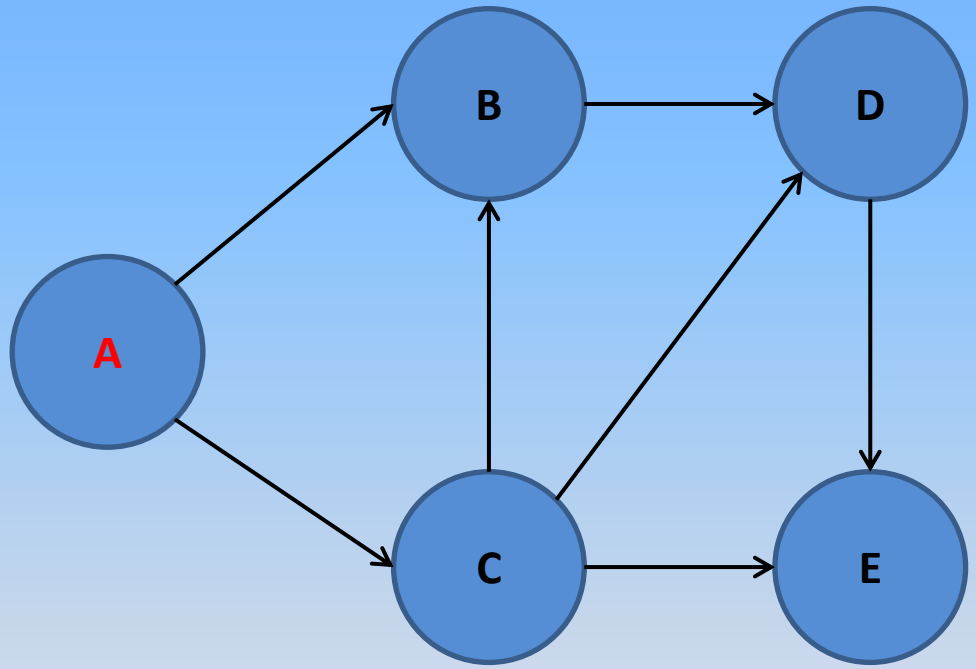
T-Sort:



Things in output stack: E, D, B

Depth First Search (DFS)

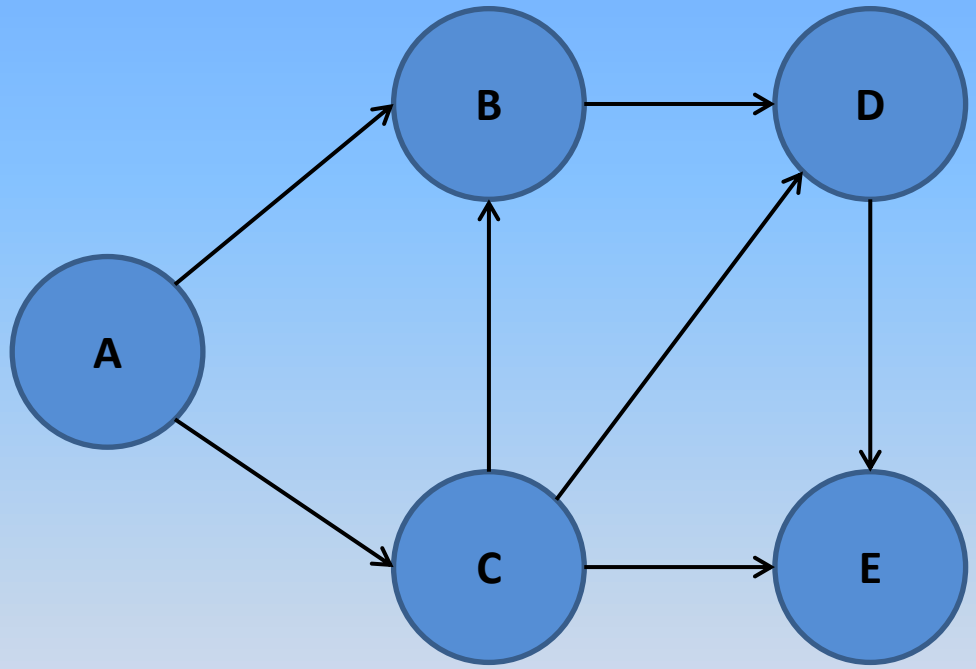
T-Sort:



Things in output stack: E, D, B, C

Depth First Search (DFS)

T-Sort:



Things in output stack: E, D, B, C, A

T-Sort Output: A, C, B, D, E

Depth First Search (DFS)

- **Strongly Connected Components**
  - **Directed Graph**
  - **Find groups of nodes such that in each group, every node has an edge to every other node**

Depth First Search (DFS)

- **Strongly Connected Components**
  - **Method: DFS twice**
  - **Do DFS on Graph G, record finishing time of node**
  - **Do DFS on Graph G^T, by decreasing finish time**
  - **Output the vertices**

Depth First Search (DFS)

- **Strongly Connected Components**
- **Note: After grouping the vertices, the new nodes form a Directed Acyclic Graph**

- **Problem: Where's the back button**

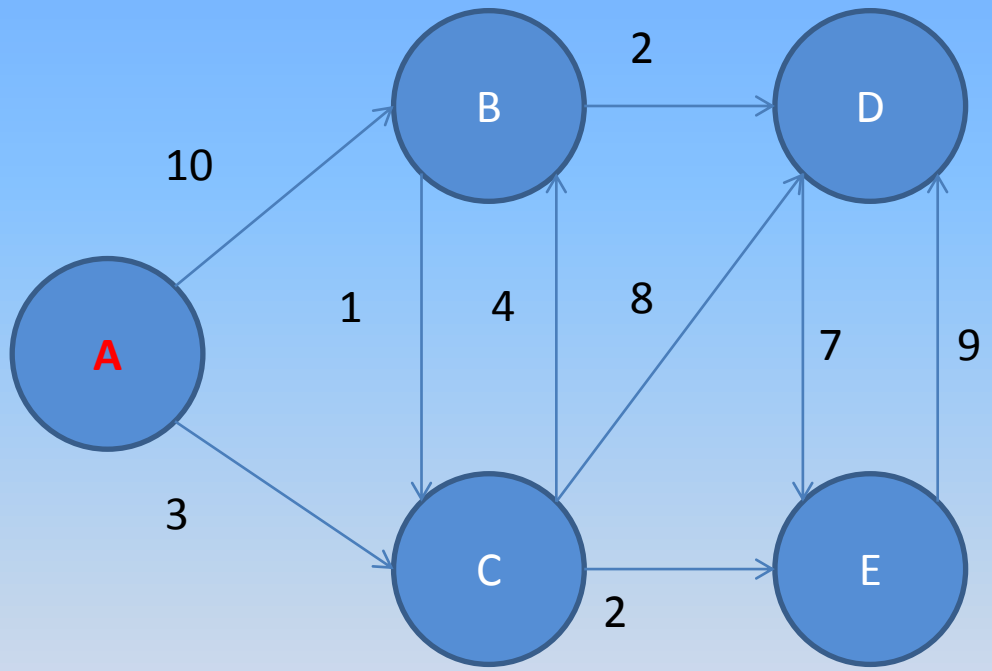
Breadth First Search (BFS)

- **Structure to use: Queue**

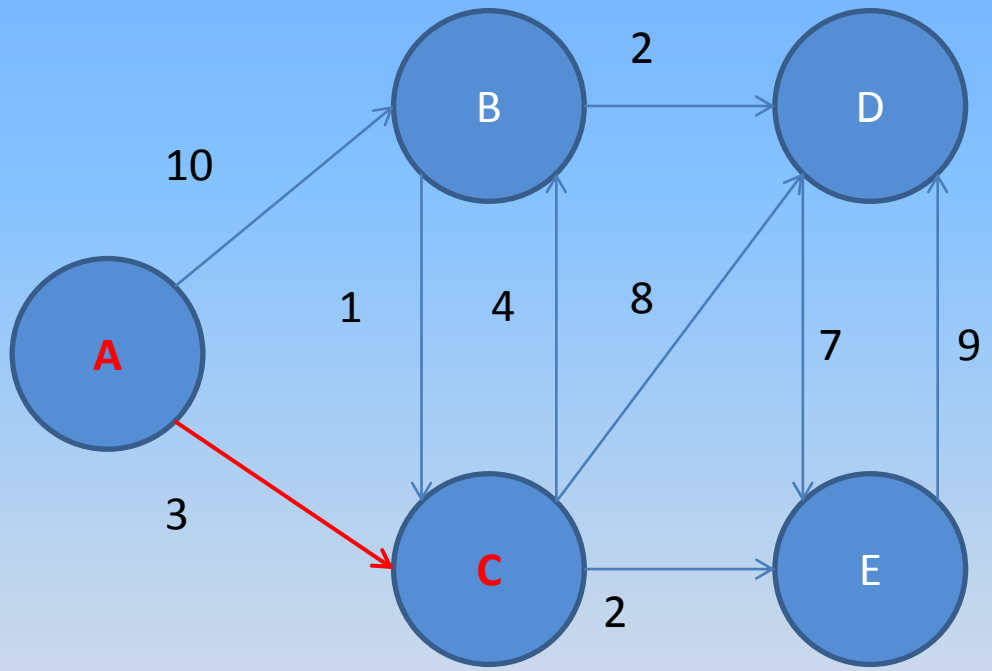
Breadth First Search (BFS)

Things in queue:

A, C	3
A, B	10



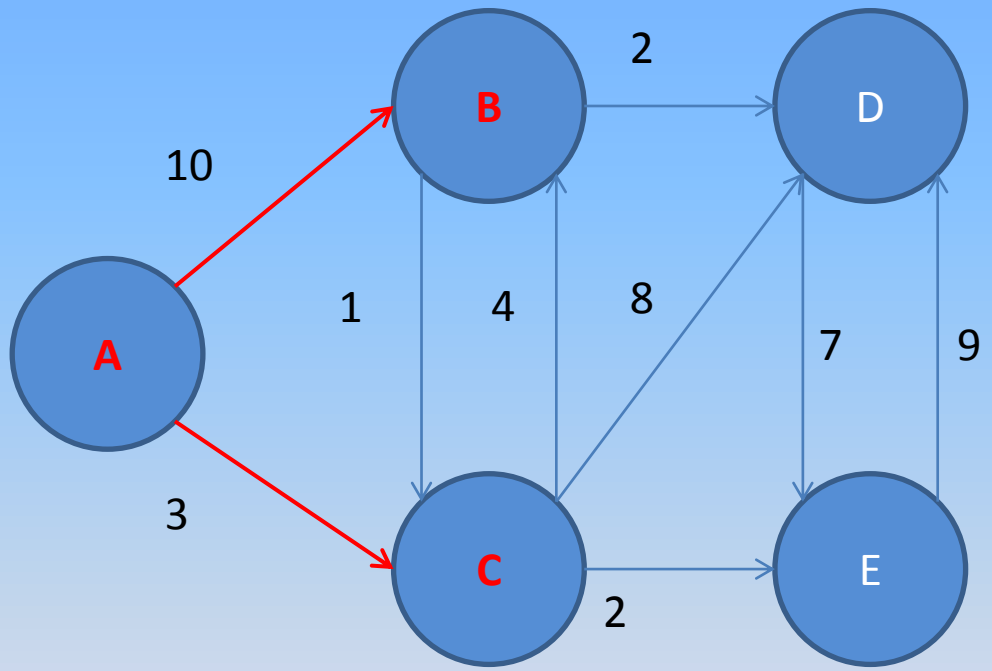
Breadth First Search (BFS)



Things in queue:

A, B	10
A, C, E	5
A, C, B	7
A, C, D	8

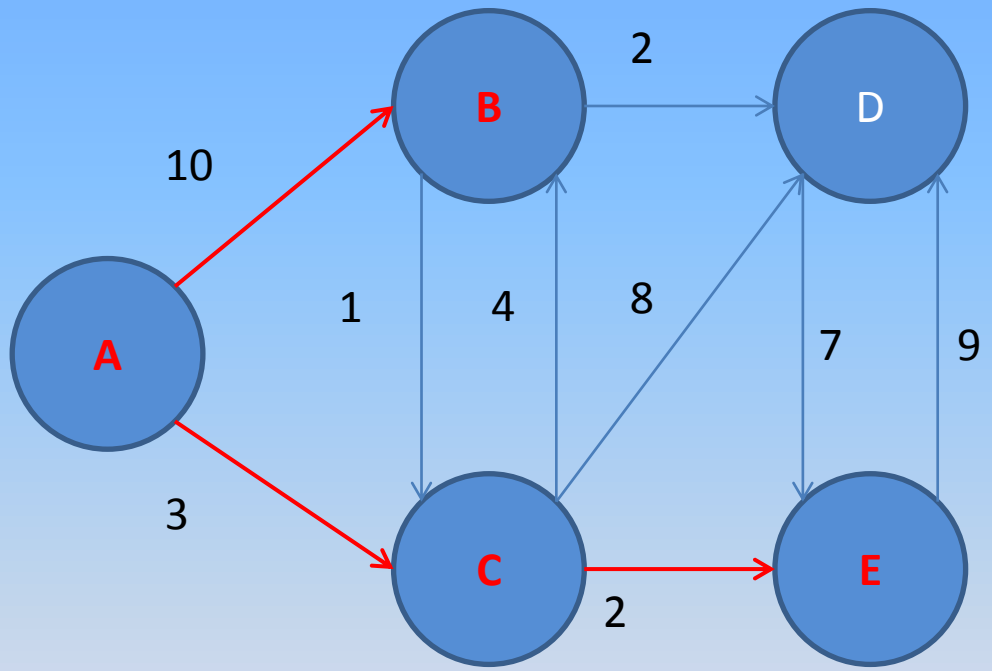
Breadth First Search (BFS)



Things in queue:

A, C, E	5
A, C, B	7
A, C, D	8
A, B, C	11
A, B, D	12

Breadth First Search (BFS)



Things in queue:

A, C, B	7
A, C, D	8
A, B, C	11
A, B, D	12

We are done !!!

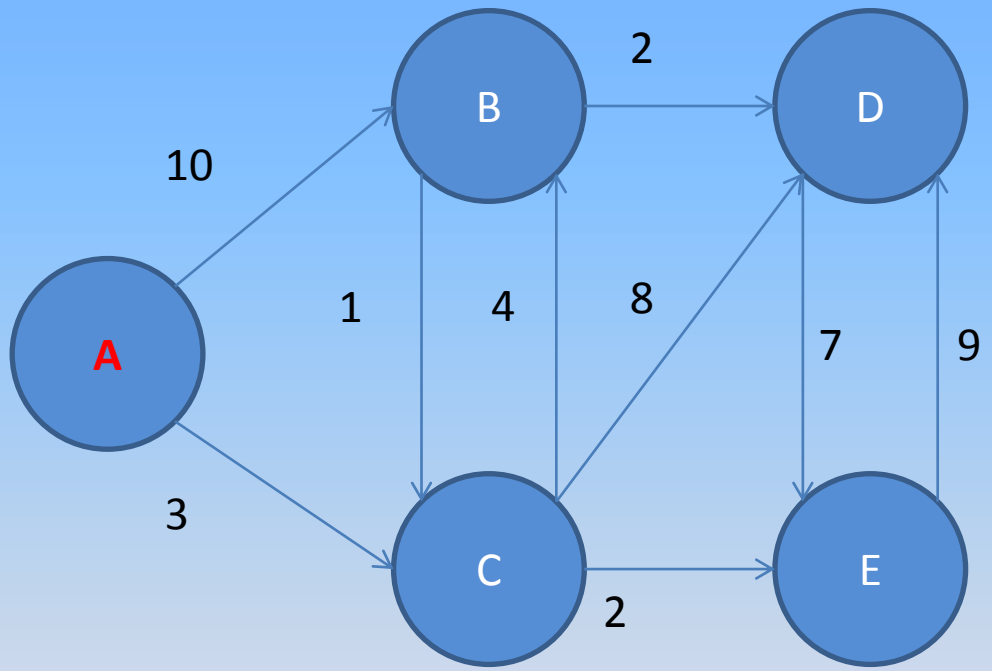
Breadth First Search (BFS)

- **Application:**
  - **Finding shortest path**
  - **Flood-Fill**

Dijkstra

- **Structure to use: Priority Queue**
- **Consider the past**
  - **The length of path visited so far**
- **No negative edges allowed**

Dijkstra



Things in priority queue:

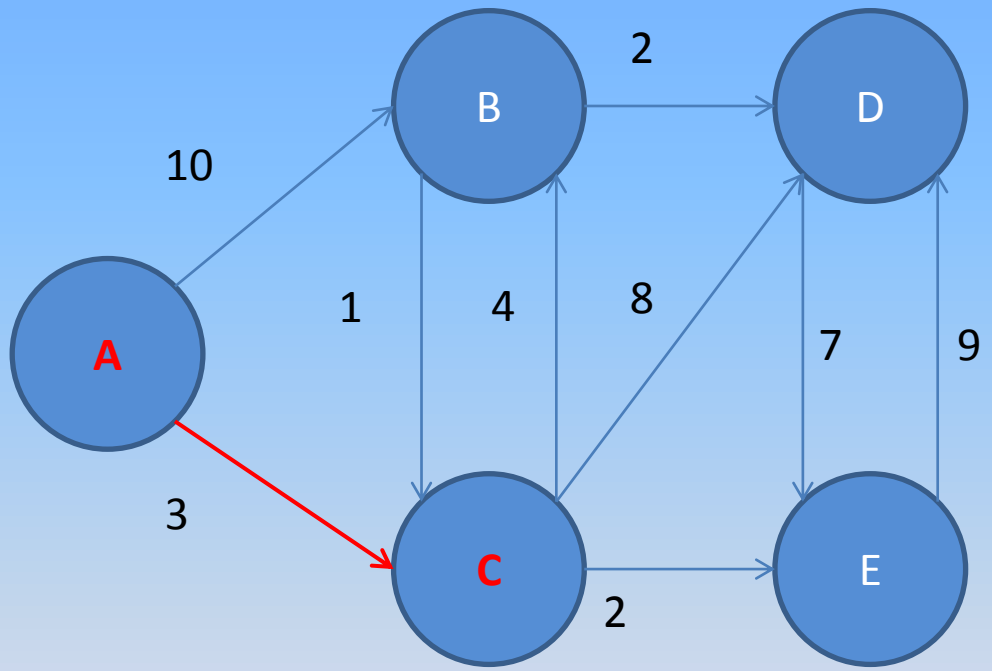
A, C 3

A, B 10

Current Route:

A

Dijkstra



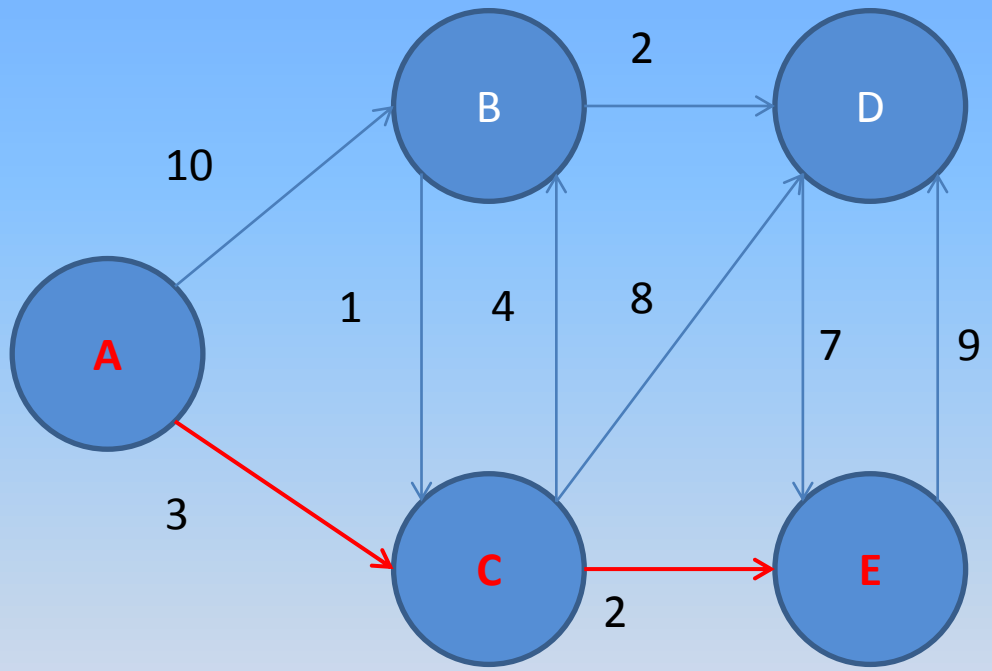
Things in priority queue:

A, C, E	5
A, C, B	7
A, B	10
A, C, D	11

Current Route:

A, C

Dijkstra



Things in priority queue:

A, C, B	7
A, B	10
A, C, D	11

Current Route:

A, C, E

We find the shortest path already !!!

A* Search

- **Structure to use: Priority Queue**
- **Consider the past + future**
- **How to consider the future?**
  - **Admissible Heuristic (Best-Case Prediction)**
  - **How to predict?**

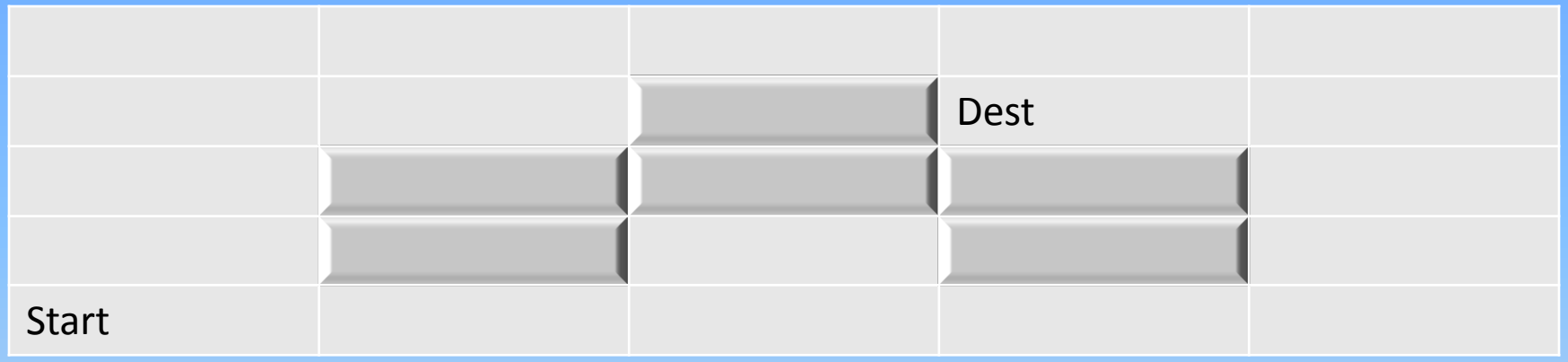
A* Search

- **Prediction 1: Displacement**
  - **Given coordinates, calculate the displacement of current node to destination**
  - **$\text{sqrt}((x2 - x1)^2 + (y2 - y1)^2)$**
- **Prediction 2: Manhattan Distance**
  - **Given grid, calculate the horizontal and vertical movement**
  - **$(x2 - x1) + (y2 - y1)$**

A* Search

- **Prediction 3: Hamming Distance**
  - **Given pieces and location, find out the number of misplaced pieces**

A* Search



- Manhattan Distance is used
- If there is a tie, consider the one with lowest heuristic first
- If there is still a tie, consider in Up, Down, Left, Right order

A* Search

			Dest	
1 + 5 = 6				
Start (0 + 6 = 6)	1 + 5 = 6			

- Manhattan Distance is used
- If there is a tie, consider the one with lowest heuristic first
- If there is still a tie, consider in Up, Down, Left, Right order

A* Search

			Dest	
2 + 4 = 6				
1 + 5 = 6				
Start (0 + 6 = 6)	1 + 5 = 6			

- Manhattan Distance is used
- If there is a tie, consider the one with lowest heuristic first
- If there is still a tie, consider in Up, Down, Left, Right order

A* Search

3 + 3 = 6			Dest	
2 + 4 = 6				
1 + 5 = 6				
Start (0 + 6 = 6)	1 + 5 = 6			

- Manhattan Distance is used
- If there is a tie, consider the one with lowest heuristic first
- If there is still a tie, consider in Up, Down, Left, Right order

A* Search

4 + 4 = 8				
3 + 3 = 6	4 + 2 = 6		Dest	
2 + 4 = 6				
1 + 5 = 6				
Start (0 + 6 = 6)	1 + 5 = 6			

- Manhattan Distance is used
- If there is a tie, consider the one with lowest heuristic first
- If there is still a tie, consider in Up, Down, Left, Right order

A* Search

4 + 4 = 8	5 + 3 = 8			
3 + 3 = 6	4 + 2 = 6		Dest	
2 + 4 = 6				
1 + 5 = 6				
Start (0 + 6 = 6)	1 + 5 = 6			

- Manhattan Distance is used
- If there is a tie, consider the one with lowest heuristic first
- If there is still a tie, consider in Up, Down, Left, Right order

A* Search

4 + 4 = 8	5 + 3 = 8			
3 + 3 = 6	4 + 2 = 6		Dest	
2 + 4 = 6				
1 + 5 = 6				
Start (0 + 6 = 6)	1 + 5 = 6	2 + 4 = 6		

- Manhattan Distance is used
- If there is a tie, consider the one with lowest heuristic first
- If there is still a tie, consider in Up, Down, Left, Right order

A* Search

4 + 4 = 8	5 + 3 = 8			
3 + 3 = 6	4 + 2 = 6		Dest	
2 + 4 = 6				
1 + 5 = 6		3 + 3 = 6		
Start (0 + 6 = 6)	1 + 5 = 6	2 + 4 = 6	3 + 3 = 6	

- Manhattan Distance is used
- If there is a tie, consider the one with lowest heuristic first
- If there is still a tie, consider in Up, Down, Left, Right order

A* Search

4 + 4 = 8	5 + 3 = 8			
3 + 3 = 6	4 + 2 = 6		Dest	
2 + 4 = 6				
1 + 5 = 6		3 + 3 = 6		
Start (0 + 6 = 6)	1 + 5 = 6	2 + 4 = 6	3 + 3 = 6	

- Manhattan Distance is used
- If there is a tie, consider the one with lowest heuristic first
- If there is still a tie, consider in Up, Down, Left, Right order

A* Search

4 + 4 = 8	5 + 3 = 8			
3 + 3 = 6	4 + 2 = 6		Dest	
2 + 4 = 6				
1 + 5 = 6		3 + 3 = 6		
Start (0 + 6 = 6)	1 + 5 = 6	2 + 4 = 6	3 + 3 = 6	4 + 4 = 8

- Manhattan Distance is used
- If there is a tie, consider the one with lowest heuristic first
- If there is still a tie, consider in Up, Down, Left, Right order

A* Search

4 + 4 = 8	5 + 3 = 8	6 + 2 = 8		
3 + 3 = 6	4 + 2 = 6		Dest	
2 + 4 = 6				
1 + 5 = 6		3 + 3 = 6		
Start (0 + 6 = 6)	1 + 5 = 6	2 + 4 = 6	3 + 3 = 6	4 + 4 = 8

- Manhattan Distance is used
- If there is a tie, consider the one with lowest heuristic first
- If there is still a tie, consider in Up, Down, Left, Right order

A* Search

4 + 4 = 8	5 + 3 = 8	6 + 2 = 8	7 + 1 = 8	
3 + 3 = 6	4 + 2 = 6		Dest	
2 + 4 = 6				
1 + 5 = 6		3 + 3 = 6		
Start (0 + 6 = 6)	1 + 5 = 6	2 + 4 = 6	3 + 3 = 6	4 + 4 = 8

- Manhattan Distance is used
- If there is a tie, consider the one with lowest heuristic first
- If there is still a tie, consider in Up, Down, Left, Right order

A* Search

4 + 4 = 8	5 + 3 = 8	6 + 2 = 8	7 + 1 = 8	8 + 2 = 10
3 + 3 = 6	4 + 2 = 6		Dest (8 + 0 = 8)	
2 + 4 = 6				
1 + 5 = 6		3 + 3 = 6		
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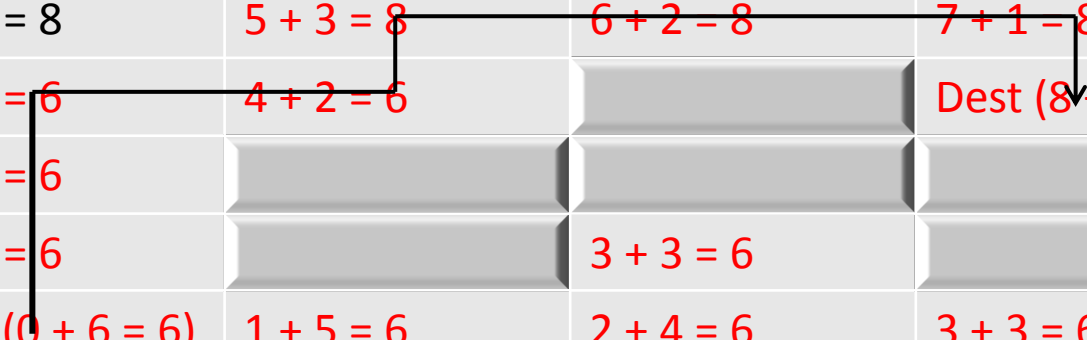


- Manhattan Distance is used
- If there is a tie, consider the one with lowest heuristic first
- If there is still a tie, consider in Up, Down, Left, Right order

A* Search

- **A* Search = Past + Future**
- **A* Search – Future = ?**
- **Dijkstra**
- **A* Search – Past = ?**
- **Greedy**

Graph Theory

- **How about graph with negative edges?**
  - **Bellman-Ford**

Bellman-Ford Algorithm

- Consider the source as weight 0, others as infinity
- Count = 0, Improve = true
- While (Count < n and Improve) {
  - Improve = false
  - Count++
  - For each edge uv
    - If u.weight + uv distance < v.weight {
      - Improve = true
      - v.weight = u.weight + uv.distance
    - }
- }
- If (Count == n) print “Negative weight cycle detected”
- Else print shortest path distance

Bellman-Ford Algorithm

- **Time Compleity: O(VE)**

Graph Theory

- **All we have just dealt with is single source**
- **How about multiple sources (all sources)?**
  - **Floyd-Warshall Algorithm**

Floyd-Warshall Algorithm

- **Makes use of memoization**
- **1. Find all smallest 1-hop distance**
- **2. Find all smallest 2-hops distance**
- **3. …**
- **N. Find all smallest n-hops distance**
- **Done !**

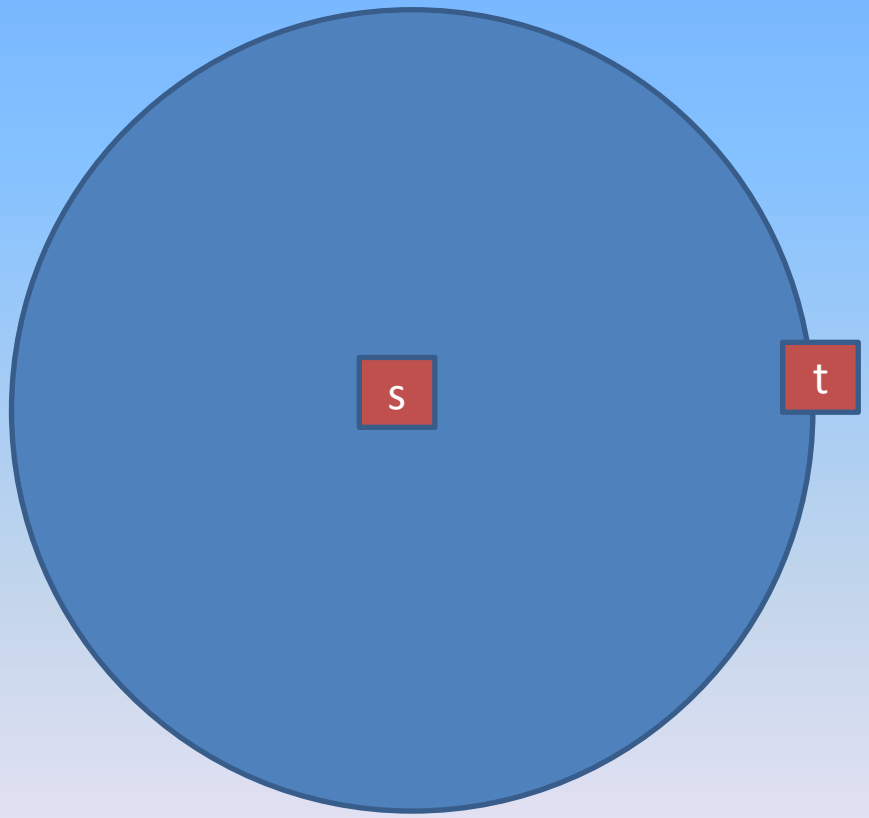
Floyd-Warshall Algorithm

- For (int k = 0; k < MAX; k++) {
  - For (int I = 0; I < MAX; i++) {
    - For (int j = 0; j < MAX; j++) {
      - Path[i][j] = min(Path[i][j], Path[i][k] + Path[k][j]);
    - }
  - }
- }

Iterative? Non-iterative?

- **What is iterative-deepening?**
- **Given limited time, find the current most optimal solution**
- **Dijkstra (Shortest 1-step Solution)**
- **Dijkstra (Shortest 2-steps Solution)**
- **Dijkstra (Shortest 3-steps Solution)**
- **…**
- **Dijkstra (Shortest n-steps Solution)**

- **Suitable in bi-directional search (Faster than 1-way search)**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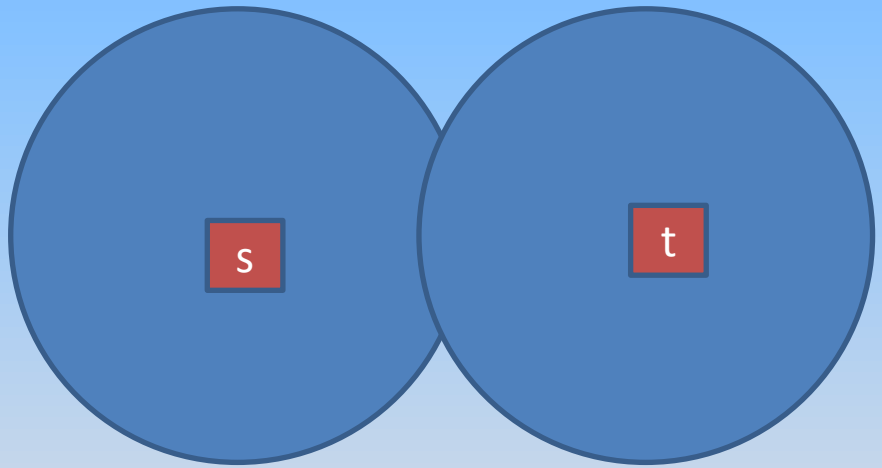
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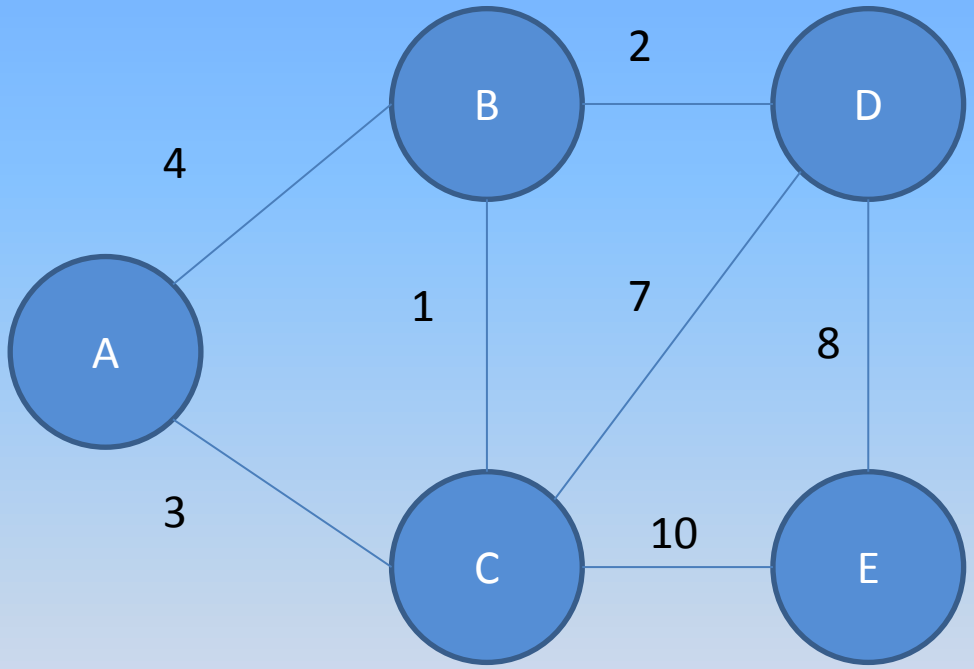
Minimum Spanning Tree (MST)

- **Given an undirected graph, find the minimum cost so that every node has path to other nodes**
- **Kruskal's Algorithm**
- **Prim's Algorithm**

Kruskal's Algorithm

- **Continue Finding Shortest Edge**
- **If no cycle is formed**
  - **add it into the list**
  - **Construct a parent-child relationship**
- **If all nodes have the same parent, we are done (path compression)**

Kruskal's Algorithm

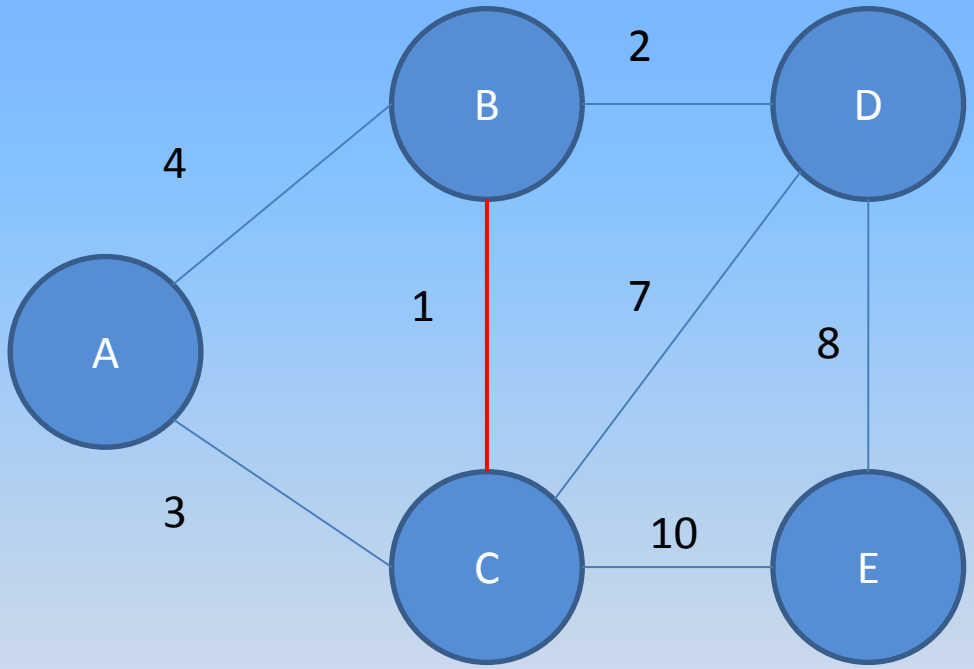


Node	A	B	C	D	E
Parent	A	B	C	D	E

Kruskal's Algorithm

Edge List:

BC 1

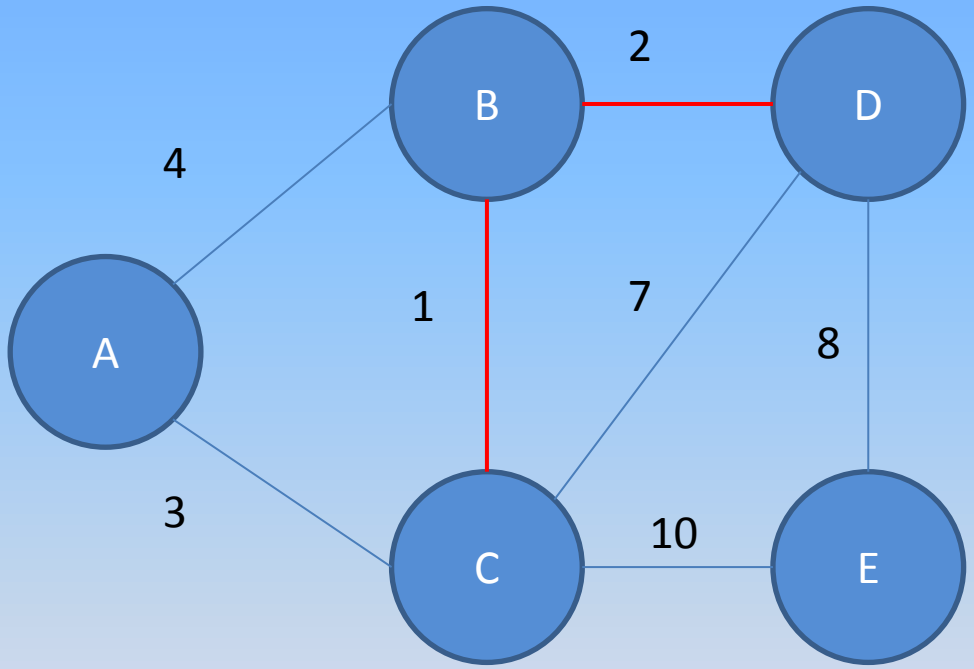


Node	A	B	C	D	E
Parent	A	B	B	D	E

Kruskal's Algorithm

Edge List:

BC 1

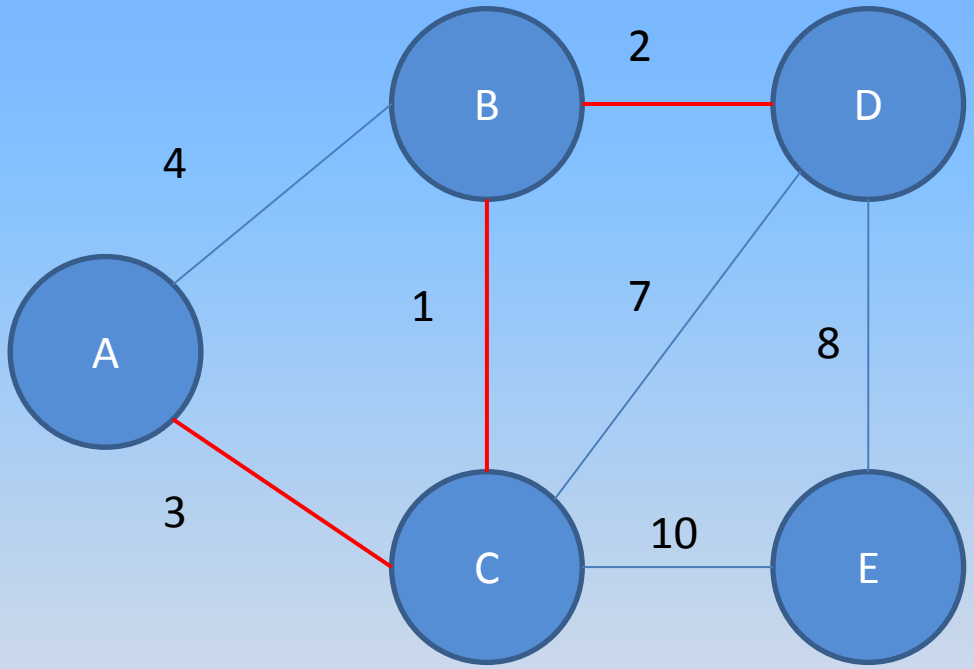


Node	A	B	C	D	E
Parent	A	B	B	B	E

Kruskal's Algorithm

Edge List:

BC 1

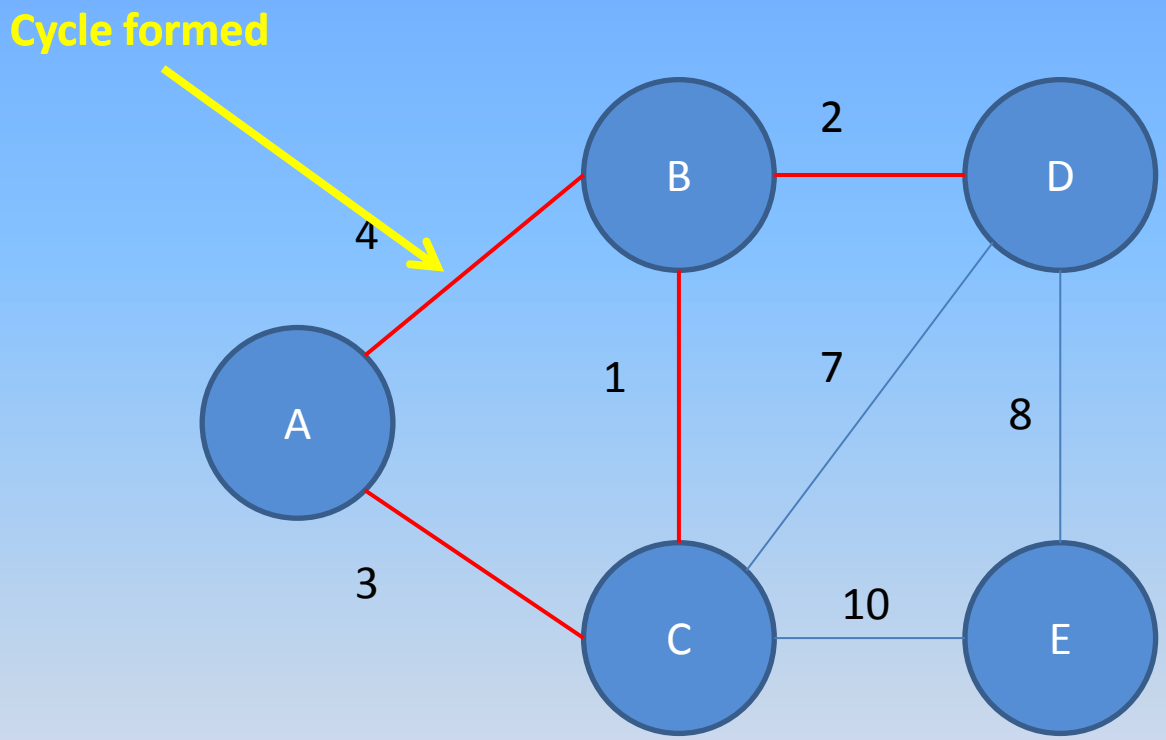


Node	A	B	C	D	E
Parent	A	A	A	A	E

Kruskal's Algorithm

Edge List:

BC 1

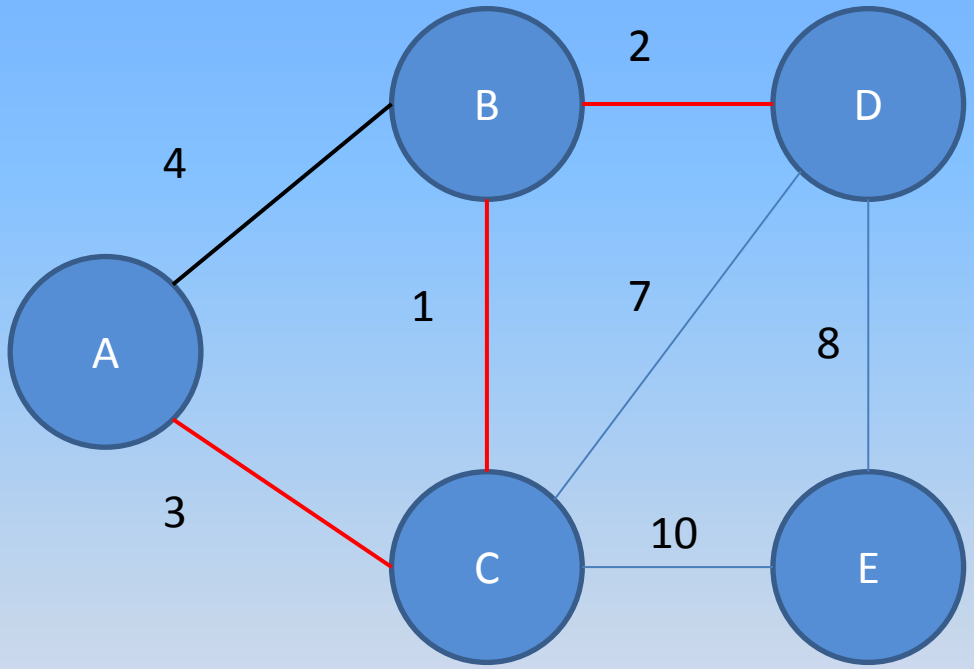


Node	A	B	C	D	E
Parent	A	A	A	A	E

Kruskal's Algorithm

Edge List:

BC 1



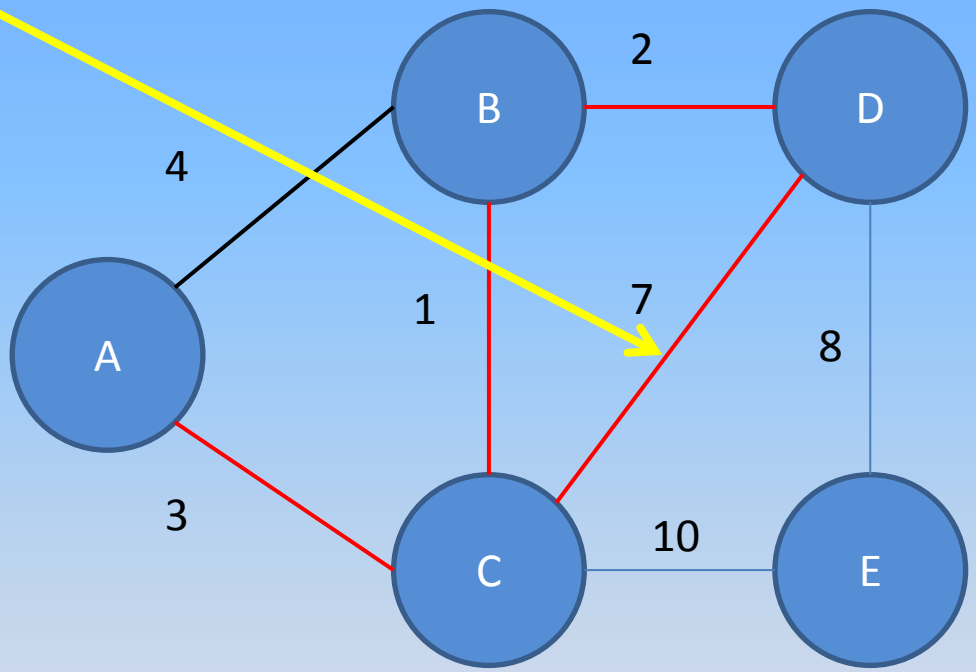
Node	A	B	C	D	E
Parent	A	A	A	A	E

Kruskal's Algorithm

Edge List:

BC 1

Cycle formed

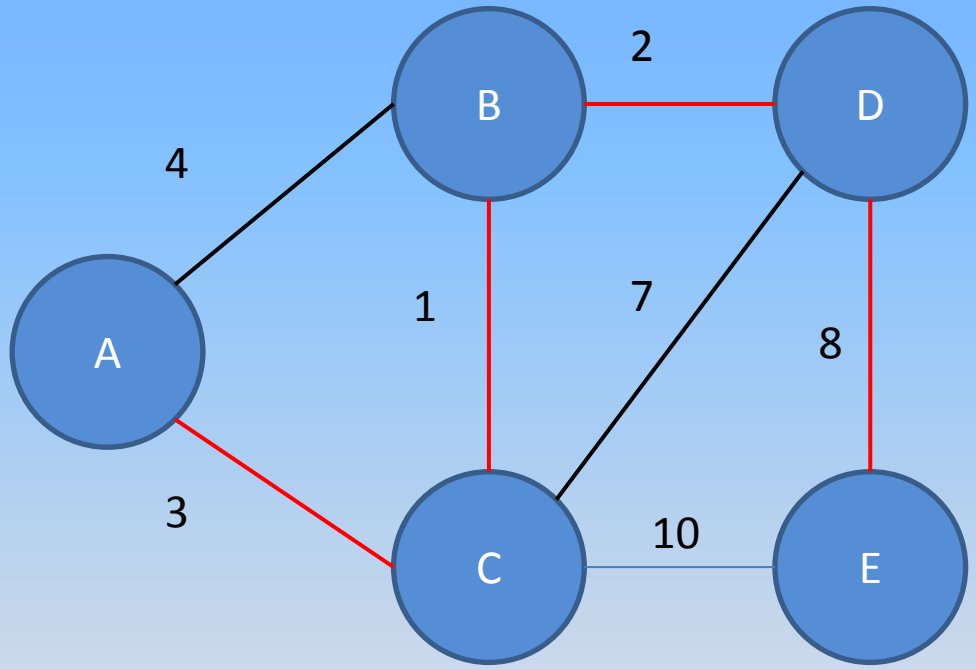


Node	A	B	C	D	E
Parent	A	A	A	A	E

Kruskal's Algorithm

Edge List:

BC 1



Node	A	B	C	D	E
Parent	A	A	A	A	A

Kruskal's Algorithm

- **Path Compression can be done when we find parent**

```
int find_parent(int x) {
    return (parent[x] == x)? x:
    (parent[x] = find_parent(parent[x]));
}
```

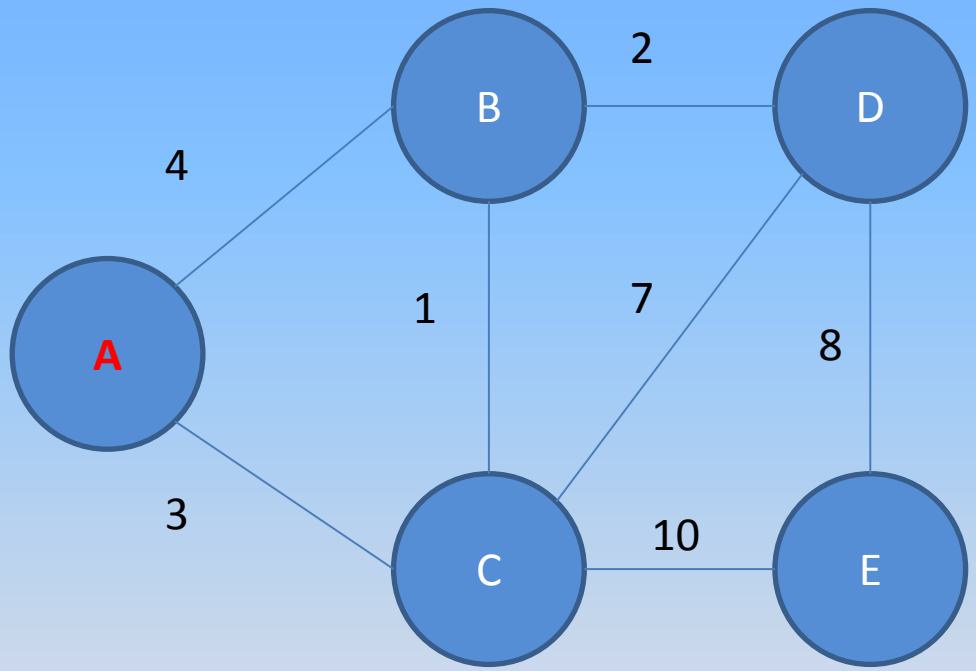
Prim's Algorithm

- **While not all nodes are visited**
  - **While the list is not empty and the minimum edge connects to visited node**
    - **Pop out the edge**
  - **If the list is empty, print Impossible and return**
  - **Else**
    - **Mark that node as visited**
    - **Add all its unvisited edges to the list**
- **Print the tree**

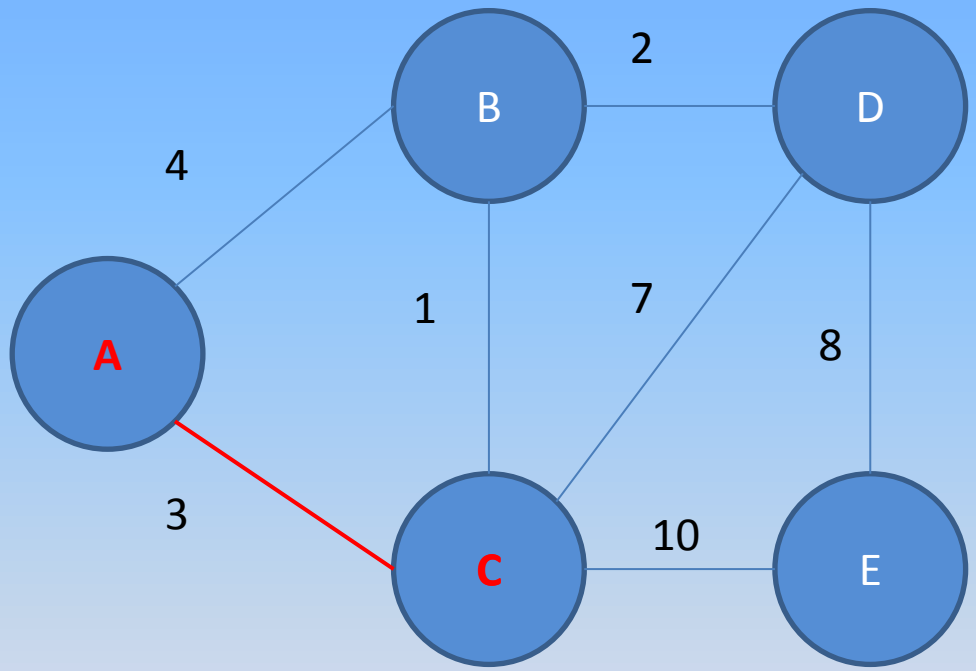
Prim's Algorithm

Edge List:

AC	3
AB	4



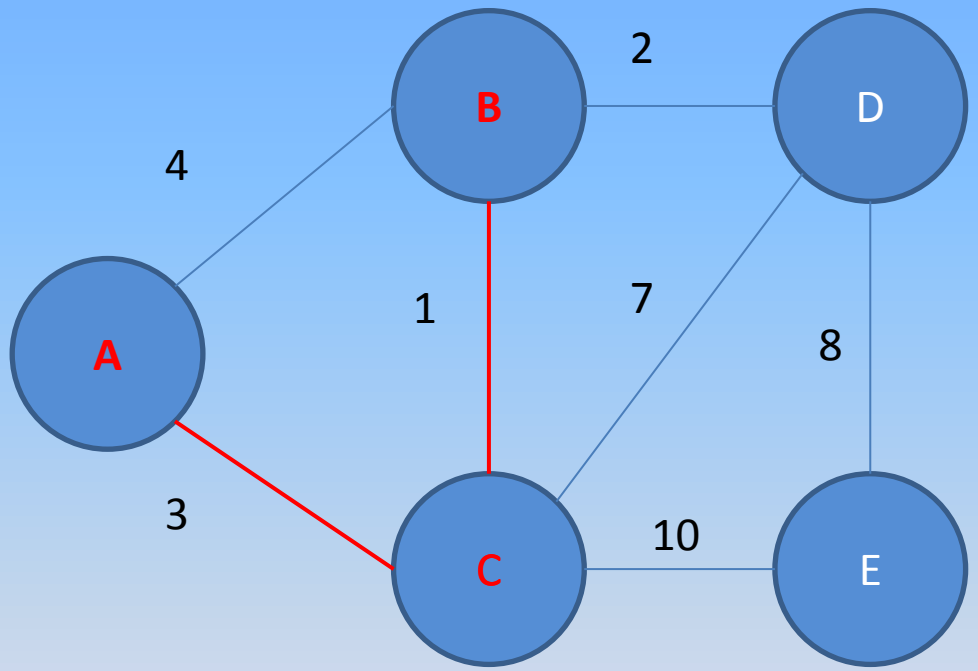
Prim's Algorithm



Edge List:

CB	1
AB	4
CD	7
CE	10

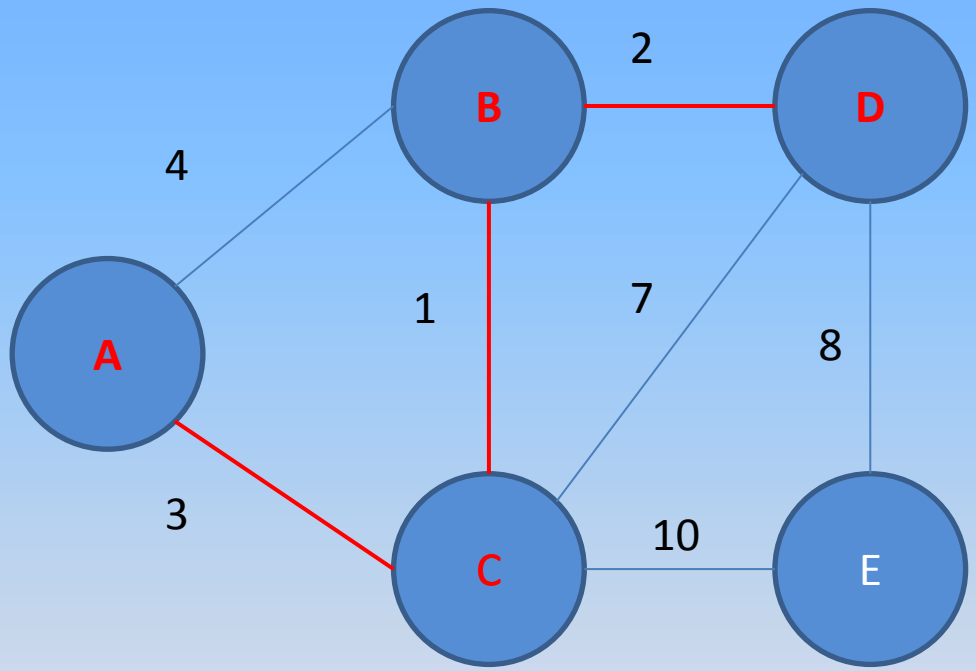
Prim's Algorithm



Edge List:

BD	2
AB	4
CD	7
CE	10

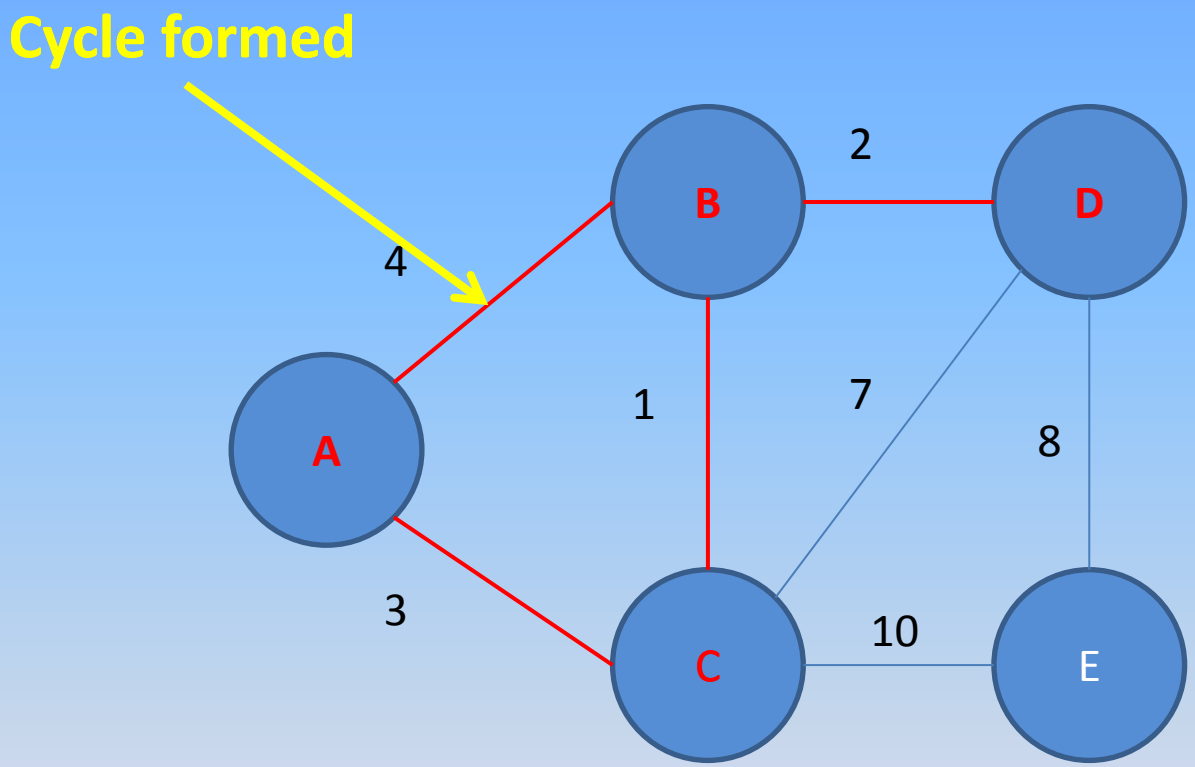
Prim's Algorithm



Edge List:

AB	4
CD	7
DE	8
CE	10

Prim's Algorithm



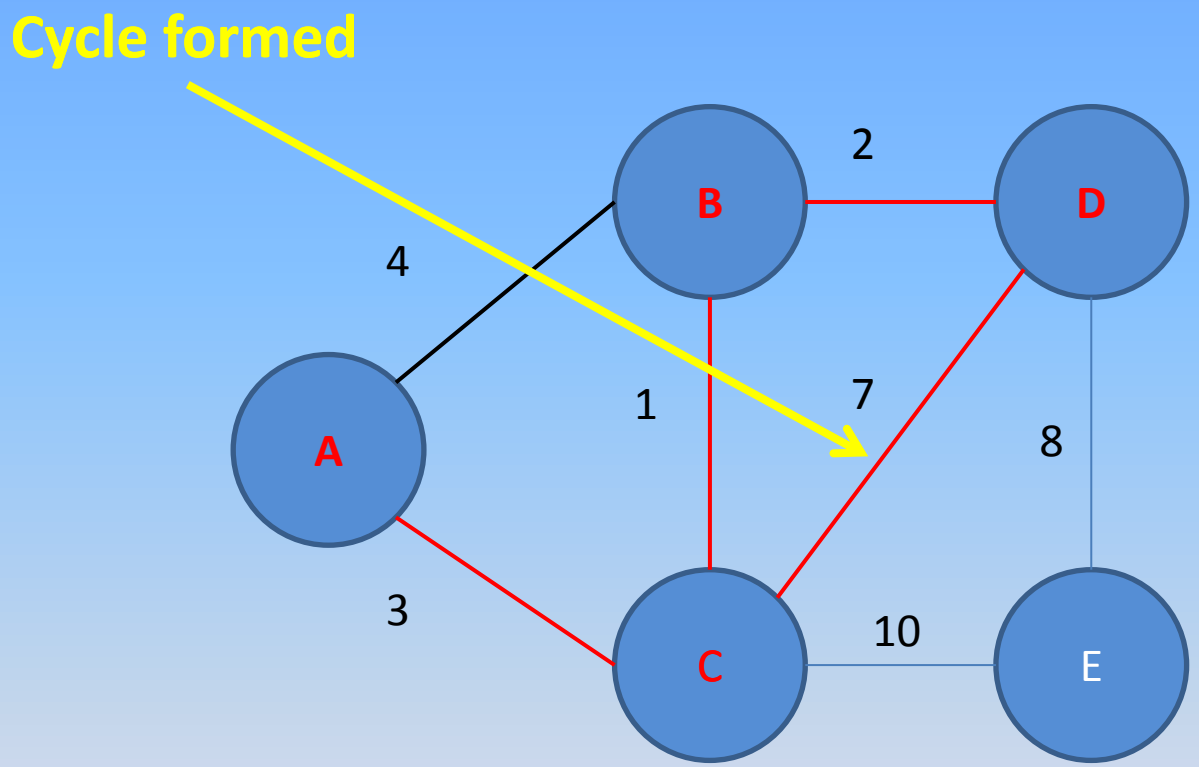
Edge List:

CD	7
DE	8
CE	10

Prim's Algorithm

Edge List:

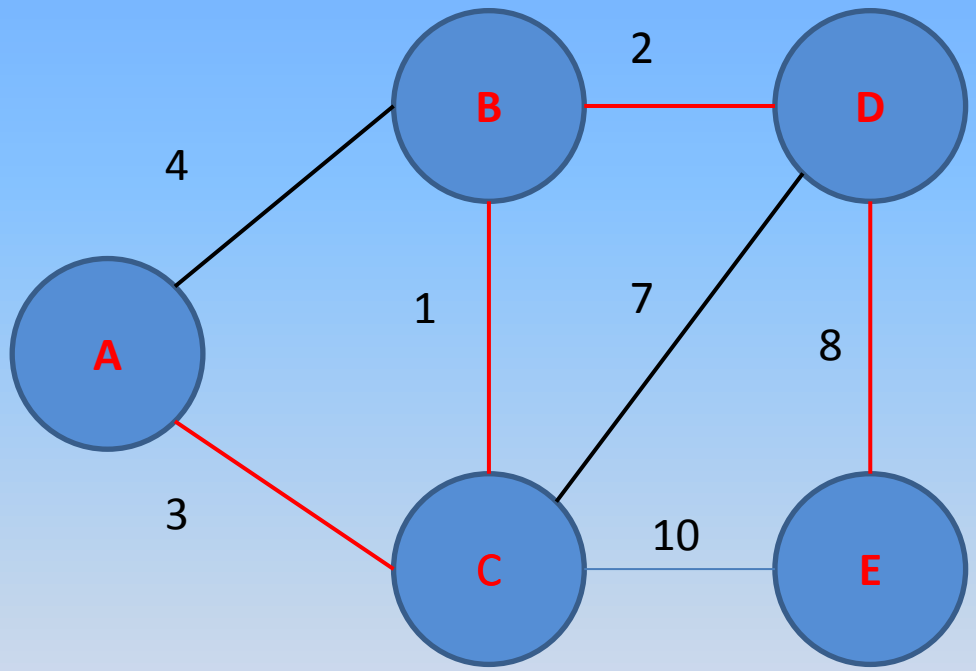
DE	8
CE	10



Prim's Algorithm

Edge List:

CE 10



We are done

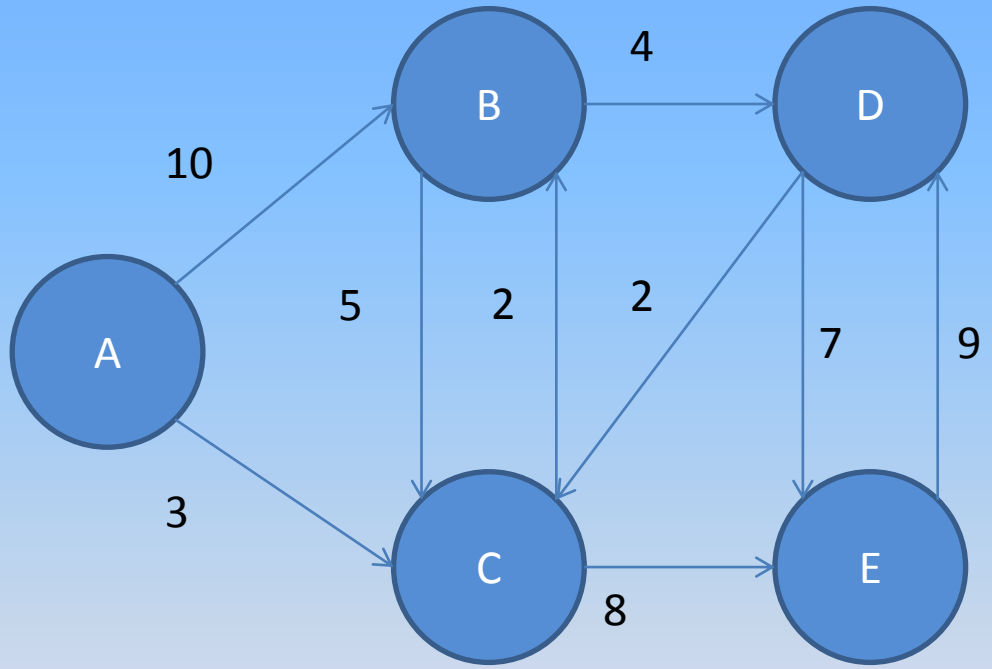
Directed Minimum Spanning Tree

- **We have just dealt with undirected MST.**
- **How about directed?**
- **Using Kruskal's or Prim's cannot solve directed MST problem**
- **How?**
- **Chu-Liu/Edmonds Algorithm**

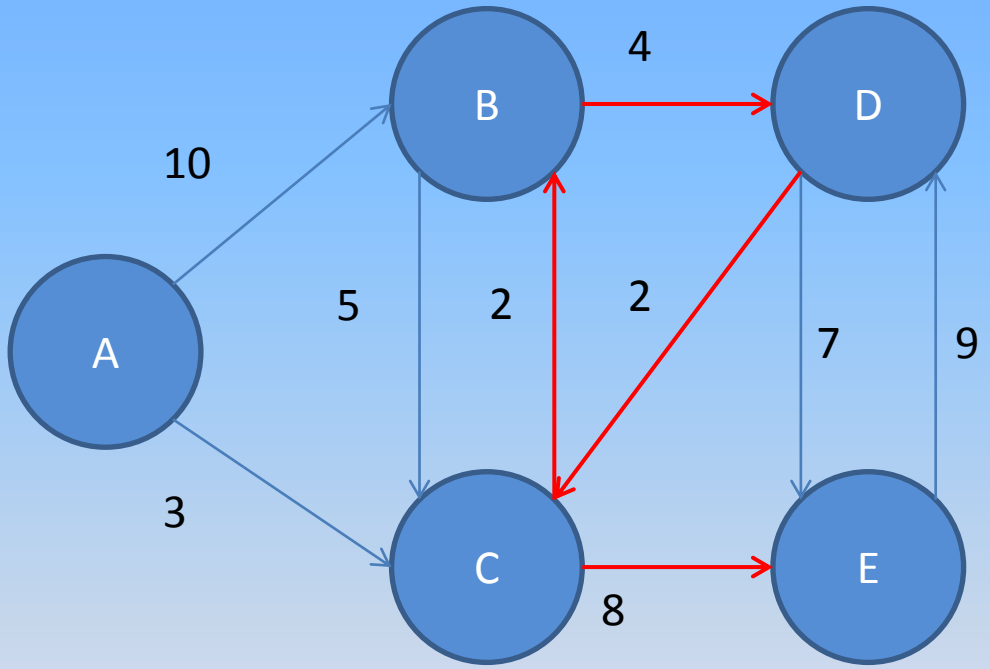
Chu-Liu/Edmonds Algorithm

- **For each vertex, find the smallest incoming edge**
- **While there is cycle**
  - **Find an edge entering the cycle with smallest increase**
  - **Remove the edge pointing to the same node and replace by new edge**
- **Print out the tree**

Chu-Liu/Edmonds Algorithm



Chu-Liu/Edmonds Algorithm



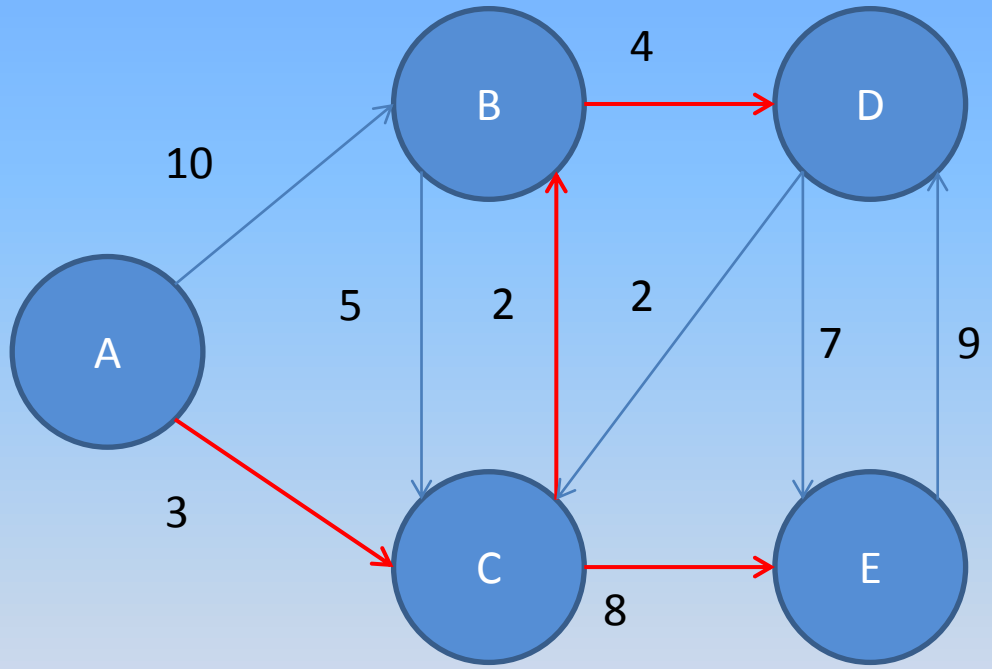
Cycle formed (B, C, D)

A → B, C, D = 10 – 2 = 8

A → B, C, D = 3 – 2 = 1

E → B, C, D = 9 – 4 = 5

Chu-Liu/Edmonds Algorithm



No Cycle formed, that's it !!!

Chu-Liu/Edmonds Algorithm

- **Using this algorithm may unloop a cycle and create another cycle…**
- **… But since the length of tree is increasing, there will have an end eventually**
- **Worst Case is doing n-1 times**
- **Time Complexity is O(EV)**

Flow

- **A graph with weight (capacity)**
- **Mission: Find out the maximum flow from source to destination**
- **How?**

Edmond-Karp

- **Do {**
  - **Do BFS on the graph to find maximum flow in the graph**
  - **Add it to the total flow**
  - **For each edge in the maximum flow**
  - **Deduct the flow just passed**
- **} While the flow is not zero;**

The End